Math 236 - Assignment 4

February 18, 2026

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. You may use technology to solve any linear systems. This assignment is due February 25.

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1. Determine whether the set is a linearly dependent or independent subset of $\mathcal{M}_{2\times 2}$.

$$\left\{ \begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}, \begin{pmatrix} 7 & -5 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 5 \\ 2 & -3 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \right\}$$

2. Determine whether the set is a linearly dependent or independent subset of $\mathcal{P}_2$.

$$\{2 + x + x^2, 1 - x + 2x^2, 3 + 2x - x^2\}$$

3. Suppose that the set $\{\vec{u}, \vec{v}, \vec{w}\}$ is a linearly independent set. Prove that $\{\vec{u}, \vec{u} + 2\vec{v}, \vec{u} + 2\vec{v} + 3\vec{w}\}$ is also a linearly independent set.

4. Determine if the following set is a basis for $\mathcal{P}_2$?

$$\{3x^2 - x + 1, 5x - 1, 6x + 1\}$$

5. Find a basis for the vector space of 2×2 symmetric matrices. Then represent the matrix A with respect to your basis.

$$A = \begin{pmatrix} 3 & 4 \\ 4 & 9 \end{pmatrix}$$

6. Represent $p(x) = 2x - x^2 + 5x^3$ with respect to the given basis for $\mathcal{P}_3$.

$$B = \{1, 1 - x, 1 + x - x^2, 1 + x + x^2 - x^3\}$$

7. A matrix is called a *Toeplitz matrix* if its diagonal entries (descending from left to right) are constant. Find a basis for, and the dimension of, the vector space of 3×3 Toeplitz matrices.

8. Write M as the span of polynomials in $\mathcal{P}_3$. Then show that your polynomials are linearly independent. What is the dimension of M?

$$M = \{a + bx + cx^2 + dx^3 : 2a + b - c - 2d = 0\}$$

9. Find a basis for, and the dimension of, the solution set of the following system.

$$\begin{array}{rcrcrcrcl} x_1 & - & 4x_2 & + & 3x_3 & - & x_4 & = & 0 \\ 2x_1 & - & 8x_2 & + & 6x_3 & - & 2x_4 & = & 0 \end{array}$$

10. Find a basis for the row space, a basis for the column space, and the rank of A.

$$A = \begin{pmatrix} 2 & 8 & -2 & -10 \\ -2 & -8 & 1 & 7 \\ -1 & -4 & 2 & 8 \\ 4 & 16 & -3 & -17 \end{pmatrix}$$