Math 236 - Assignment 6

March 4, 2026

Name ____________________

Score ______________

Show all work to receive full credit. Supply explanations when necessary. You may use technology to solve any linear systems. This assignment is due March 11.

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1. Show that $f : \mathcal{M}_{2\times 2} \to \mathbb{C}$ is a homomorphism.

$$f(\begin{pmatrix} a & b \\ c & d \end{pmatrix}) = (a + 2c) + (b - d)i$$

2. Let's make a slight change to the function defined in problem 1.

$$f(\begin{pmatrix} a & b \\ c & d \end{pmatrix}) = (a + 2) + (b - d)i$$

   Show that f is not a homomorphism.

3. Let $n : \mathbb{R}^3 \to \mathbb{R}$ be defined by

$$n(\begin{pmatrix} x \\ y \\ z \end{pmatrix}) = \sqrt{x^2 + y^2 + z^2}.$$

   Give an simple example to show that n is not a homomorphism.

4. Let $d : \mathbb{R}^3 \to \mathbb{R}$ be defined by

$$d(\begin{pmatrix} x \\ y \\ z \end{pmatrix}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \cdot (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}).$$

   Show that d is a homomorphism. (The centered dot denotes the dot product from Calculus III.)

5. Assume that $h : V \to W$ is a homomorphism. The *null space* of h is

$$\mathcal{N}(h) = \{\vec{v} \in V : h(\vec{v}) = \vec{0}_W\}.$$

   Show that the null space is a subspace of V.

6. Assume that $h : V \to W$ is a homomorphism. The *range* of h is

$$\mathcal{R}(h) = \{\vec{w} \in W : \vec{w} = h(\vec{v}) \text{ for some } \vec{v} \in V\}.$$

   Show that the range is a subspace of W.

Turn over.

7. Suppose $h : V \to V$ is a homomorphism and that $B = \langle \vec{\beta}_1, \vec{\beta}_2, \ldots, \vec{\beta}_n \rangle$ is a basis for V. Prove the statement: If $h(\vec{\beta}_i) = \vec{0}$ for each basis vector, then h is the zero map.

8. For the map $h : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$h(\begin{pmatrix} x \\ y \\ z \end{pmatrix}) = \begin{pmatrix} 2x + y \\ x - z \end{pmatrix},$$

   find the range space, rank, null space, and nullity.

9. Define $F, G : \mathcal{P}_2 \to \mathbb{R}^2$ by

$$F(ax^2 + bx + c) = \begin{pmatrix} a + c \\ b \end{pmatrix} \quad \text{and} \quad G(ax^2 + bx + c) = \begin{pmatrix} 2b \\ -3c \end{pmatrix}.$$

   Do not bother showing that F and G are both homomorphisms, but show that the linear combination $4F - 2G$ is a homomorphism from $\mathcal{P}_2$ into $\mathbb{R}^2$.