Math 236 - Assignment 8

April 1, 2026

Name ____________________

Score ________________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 8.

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1. A *permutation matrix* is a square matrix that has exactly one 1 in each row and column and 0's everywhere else. Experiment with multiplications of random matrices by permutation matrices (on either side), and try to predict their effect. Argue that the inverse of any permutation matrix is its transpose. (You need not give a formal proof, just a compelling argument.)

2. For an invertible matrix A and a positive integer k, prove that $(A^k)^{-1} = (A^{-1})^k$. (Technically, this proof requires a technique called induction. We will eventually talk about induction. For now, to complete the proof, simply write out what A^k and $(A^{-1})^k$ mean, and show that their product is I.)

3. Suppose that A and B are $n \times n$ matrices. Prove that $AB = I$ if and only if $BA = I$. (Helpful hint: From the fact that $\operatorname{rank}(AB) = \operatorname{rank}(BA) = n$, it follows that $\operatorname{rank}(A) = \operatorname{rank}(B) = n$.)

4. Find the change of basis matrix for $B, D \subseteq \mathbb{R}^2$.
$$B = \left\langle \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\rangle, \qquad D = \left\langle \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle$$

5. Find the change of basis matrix for $B, D \subseteq \mathcal{P}_1$.
$$B = \langle 1 + x, 1 - x \rangle, \qquad D = \langle 2x, 1 - 2x \rangle$$

6. Find the change of basis matrix for $B, D \subseteq \mathcal{P}_2$.
$$B = \langle 1, x, x^2 \rangle, \qquad D = \langle x^2, 1, x \rangle$$

7. Perform the Gram-Schmidt process on this basis for $\mathbb{R}^3$:
$$\left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \right\rangle.$$

8. Find an orthonormal basis for this subspace of $\mathbb{R}^4$:
$$\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : x - y - z + w = 0 \text{ and } x + z = 0 \}.$$

9. What happens if we apply the Gram-Schmidt process to a finite set that is not linearly independent?

10. What happens if we apply the Gram-Schmidt process to a basis that is already orthogonal?