

Math 236 - Assignment 9

April 8, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 15.

1. Suppose that $B = \{\vec{\beta}_1, \vec{\beta}_2, \dots, \vec{\beta}_n\}$ is a set of nonzero mutually orthogonal vectors. Prove that the set is linearly independent.
2. Use any method to compute the determinant by hand.

$$A = \begin{pmatrix} 2 & 4 & -6 \\ 5 & 1 & 3 \\ 2 & -6 & 1 \end{pmatrix}$$

3. Use Gaussian elimination to compute the determinant by hand.

$$B = \begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

4. Use Cramer's rule to solve the following system of equations.

$$\begin{aligned} 7x + 3y &= 12 \\ 3x - 9y &= 2 \end{aligned}$$

5. Determine all real numbers x that make this matrix singular?

$$\begin{pmatrix} 1-x & 3 & 3 \\ -3 & -5-x & -3 \\ 3 & 3 & 1-x \end{pmatrix}$$

6. Let $A = \begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix}$. Find all real or complex numbers x that make the matrix $A - xI$ singular.

7. For which values of k does this system have a unique solution?

$$\begin{aligned} x &+ z - w &= 2 \\ &y - 2z &= 3 \\ x &+ kz &= 4 \\ &z - w &= 2 \end{aligned}$$

8. What is the determinant of each kind of elementary matrix?

Turn over.

9. Use the Laplace expansion to compute the determinant by hand. Expand over whichever row or column is most convenient.

$$\begin{pmatrix} 1 & 5 & 0 \\ 2 & 1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

10. Find the inverse by using the matrix adjoint.

$$\begin{pmatrix} 1 & 4 & 3 \\ -1 & 0 & 4 \\ 1 & 8 & 9 \end{pmatrix}$$