Math 236 - Test 1

February 11, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. Unless otherwise indicated, you may use your calculator to obtain any RREF. Write all solution sets in our standard vector form.

1. (6 points) State the three elementary row operations.

2. (10 points) Reduce the system to echelon form (but not RREF). Then find the solution set by backsolving. Do not use your calculator for the reduction. Indicate which row operations you used.

$$
\begin{array}{rcrcrcrcr}
2x &+& 7y &+& 2z &+& w &=& 21 \\
x &+& 2y &+& z &-& w &=& 5 \\
&& 3y &&&+& 2w &=& 7
\end{array}
$$

3. (4 points) The following system is in echelon form. State the leading variable(s) and the free variable(s).

$$
\begin{aligned}
x \quad - \quad 3z \quad + \quad 5w &= \quad 1 \\
4y \qquad\qquad\qquad &= \quad -9 \\
6w &= \quad 3
\end{aligned}
$$

4. (6 points) Consider the linear system

$$
\begin{aligned}
2x \quad - \quad 4y &= \quad k \\
-x \quad + \quad ay &= \quad 5
\end{aligned}
$$

where a and k are real numbers. Find all values of a and k for which the system has

(a) no solution.

(b) infinitely many solutions.

(c) a unique solution.

5. (8 points) Write the system as an augmented matrix. Then compute the RREF and determine the complete solution set. You may use your calculator to find the RREF.

$$
\begin{array}{rrrrrrr}
35x_1 & + & 7x_2 & + & 2x_3 & - & 50x_4 & - & 54x_5 & = & 154 \\
5x_1 & + & x_2 & & & - & 10x_4 & - & 9x_5 & = & 30 \\
-10x_1 & - & 2x_2 & & & + & 25x_4 & + & 20x_5 & = & -75
\end{array}
$$

6. (8 points) According to our definition from class, what does it mean for a matrix to be nonsingular? Use our definition to show that the following matrix is singular. You may use your calculator as you see fit.

$$
\begin{pmatrix}
1 & 2 & 1 \\
4 & 1 & 0 \\
3 & -1 & -1
\end{pmatrix}
$$

7. (10 points) On the set of positive real numbers, we will say $x \sim y$ if and only if $x - y$ is an integer. Prove that $\sim$ is an equivalence relation.

8. (4 points) For real numbers a and b, we will say $a \sim b$ if and only if $|a - b| \leq 1$. Show that $\sim$ is NOT an equivalence relation.

9. (4 points) Give an example of a matrix that is row equivalent to A and say how you know.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

10. (5 points) Using any of the facts we know from class, explain why any 2×2 nonsingular matrix must be row equivalent to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

11. (4 points) A set W is *closed under linear combinations* if, for any scalars α and β and for any x and y in W, the linear combination $\alpha x + \beta y$ is also in W. Show that the following set is closed under linear combinations.

$$W = \{p(x) \in \mathcal{P}_2 : p(x) = 0\}$$

12. (5 points) Recall that in a vector space, the vectors x and y are called additive inverses if their sum is the zero vector. Every vector in a vector space must have an additive inverse. Prove that each vector in a vector space has a unique additive inverse.

13. (10 points) Consider the set S of all 2×2 symmetric matrices with the usual matrix addition and scalar multiplication:

$$S = \left\{ \begin{pmatrix} a & b \\ b & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}.$$

S is a vector space. Prove any five of the vector space properties.

14. (4 points) Look back at the problem above, but now let N be the set of all 2×2 **nonsymmetric** matrices (with the usual operations). Show that N is NOT closed under addition.

15. (12 points) Consider the following set P as a subset of $\mathcal{P}_3$ with the usual operations of polynomial addition and multiplication by a constant.

$$P = \{ax^3 + bx^2 + cx + d : a + 2b + c = 0 \text{ and } b + c + d = 0\}$$

(a) Show that P is closed under scalar multiplication.

(b) Show that any element of P has an additive inverse in P.

(c) Write P as the span of two polynomials in $\mathcal{P}_3$.

(d) Is P a subspace of $\mathcal{P}_3$? How do you know?