Math 240 - Quiz 1

August 27, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due on September 1.

1. (2 points) Is the following ordinary differential equation linear or nonlinear? Briefly explain your reasoning. Then show that $y = 1/(1+x^2)$ is a solution.

$$y' + 2xy^2 = 0$$

THE EQUATION IS NONLINEAR BECAUSE OF THE y^2. $y' + 2xy = 0$ IS LINEAR

$y' + 2xy^2 = 0$ IS NOT LINEAR.

$$y' = \frac{dy}{dx} = \frac{d}{dx}\frac{1}{1+x^2} = \frac{-2x}{(1+x^2)^2}$$

$$\Rightarrow \quad y' + 2xy^2 = \frac{-2x}{(1+x^2)^2} + 2x\left(\frac{1}{1+x^2}\right)^2 = \frac{-2x}{(1+x^2)^2} + \frac{2x}{(1+x^2)^2} = 0 \checkmark$$

2. (3 points) Is the following ordinary differential equation linear or nonlinear? Briefly explain your reasoning. Then verify that $y = Ce^{-x^3}$ is a solution for any constant C. Finally, determine the constant C so that $y(0) = 7$.

$$y' + 3x^2y = 0$$

THE EQUATION IS LINEAR. IT HAS THE general FORM OF A 1^{ST} DEGREE LINEAR EQUATION: $a(x)y' + b(x)y = c(x)$.

$$y = Ce^{-x^3} \Rightarrow y' = \frac{dy}{dx} = -3x^2Ce^{-x^3}$$

$$\Rightarrow \quad y' + 3x^2y = -3x^2Ce^{-x^3} + 3x^2Ce^{-x^3} = 0 \checkmark$$

$$y(0) = 7 \Rightarrow Ce^0 = 7 \Rightarrow C = 7$$

$$\boxed{y = 7e^{-x^3}}$$

3. (2 points) Read the problem situation below. Write a differential equation having $y = g(x)$ as one of its solutions.

The line tangent to the graph of g at (x, y) passes through the point $(x/2, 0)$.

TANGENT LINE PASSES THROUGH (x,y) & $(x/2,0)$

$\Rightarrow$ SLOPE OF TAN LINE $= \dfrac{y-0}{x-\frac{x}{2}} = \dfrac{y}{x/2}$

$$\frac{dy}{dx} = \frac{2y}{x}$$

4. (3 points) Solve the initial value problem: $\underbrace{\dfrac{dy}{dx} = x\sqrt{x^2+9},\ y(-4) = 0}$

HAS THE FORM $\dfrac{dy}{dx} = f(x)$.

SOLVE BY DIRECT INTEGRATION.

$$y(x) = 0 + \int_{-4}^{x} t\sqrt{t^2+9}\,dt = \frac{1}{2}\int_{25}^{x^2+9} u^{1/2}\,du$$

$u = t^2+9$

$du = 2t\,dt$

$$= \frac{1}{3}u^{3/2}\Big|_{u=25}^{u=x^2+9}$$

$$y(x) = \frac{1}{3}(x^2+9)^{3/2} - \frac{125}{3}$$