Math 240 - Quiz 2
September 3, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due on September 8.

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1. (4 points) Consider the differential equation $\frac{dy}{dx} = xy^{1/2}$, $y \geq 0$.

   (a) Referring to our existence/uniqueness theorem, explain why we should not expect this ODE to have a unique solution passing through any point where $y = 0$.

$f(x,y) = xy^{1/2}$. f IS CONTINUOUS ON $-\infty < x < \infty$, $0 \leq y < \infty$

$\hookrightarrow$ A SOLUTION IS GUARANTEED.

$f_y(x,y) = \frac{1}{2}xy^{-1/2}$. f_y IS CONT. ON $-\infty < x < \infty$, $0 < y < \infty$

$\hookrightarrow$ SINCE f_y IS NOT CONT AT $y=0$, A UNIQUE SOL'N IS NOT GUARANTEED.

   (b) Use a slope field generator (see the links on our Lecture Resources page) to construct the slope field for the ODE. How many solutions pass through $(0,0)$?

DEPENDING ON THE RESOLUTION USED IN PLOTTING THE SLOPE FIELD, YOU MAY SEE 1 OR 2 POSSIBLE SOLUTIONS.

SEE ATTACHED SHEET.

   (c) Find the solutions passing through $(0,0)$.

$$\frac{dy}{y^{1/2}} = x\,dx$$

$$2y^{1/2} = \frac{1}{2}x^2 + C$$

$$y^{1/2} = \frac{1}{4}x^2 + C$$

$$y = \left(\frac{1}{4}x^2 + C\right)^2$$

$$y(0) = 0 \Rightarrow C = 0$$

$$y(x) = \frac{x^4}{16}$$

ANOTHER SOLUTION IS THE CONSTANT FUNCTION $y(x) = 0$

2. (3 points) Solve the initial value problem: $\dfrac{dy}{dx} = xy^3 + 3x^2y^3, \quad y(1) = 1/2.$

$$\frac{dy}{dx} = y^3(x + 3x^2) \Rightarrow \frac{1}{y^3}\,dy = (x + 3x^2)\,dx$$

$$\int y^{-3}\,dy = \int (x + 3x^2)\,dx$$

$$-\frac{1}{2y^2} = \frac{1}{2}x^2 + x^3 + C_1$$

$$\frac{1}{y^2} = -x^2 - 2x^3 + C_2$$

$$y^2 = \frac{1}{C_2 - x^2 - 2x^3} \Rightarrow y(x) = \frac{1}{\sqrt{C - x^2 - 2x^3}}$$

(POS. SQUARE ROOT BASED ON IC)

$$y(1) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{\sqrt{C-3}}$$

$$C = 7$$

$$y(x) = \frac{1}{\sqrt{7 - x^2 - 2x^3}}$$

3. (3 points) Solve the initial value problem: $x\dfrac{dy}{dx} = 1 + x + y, \quad y(1) = 4.$

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{1+x}{x}$$

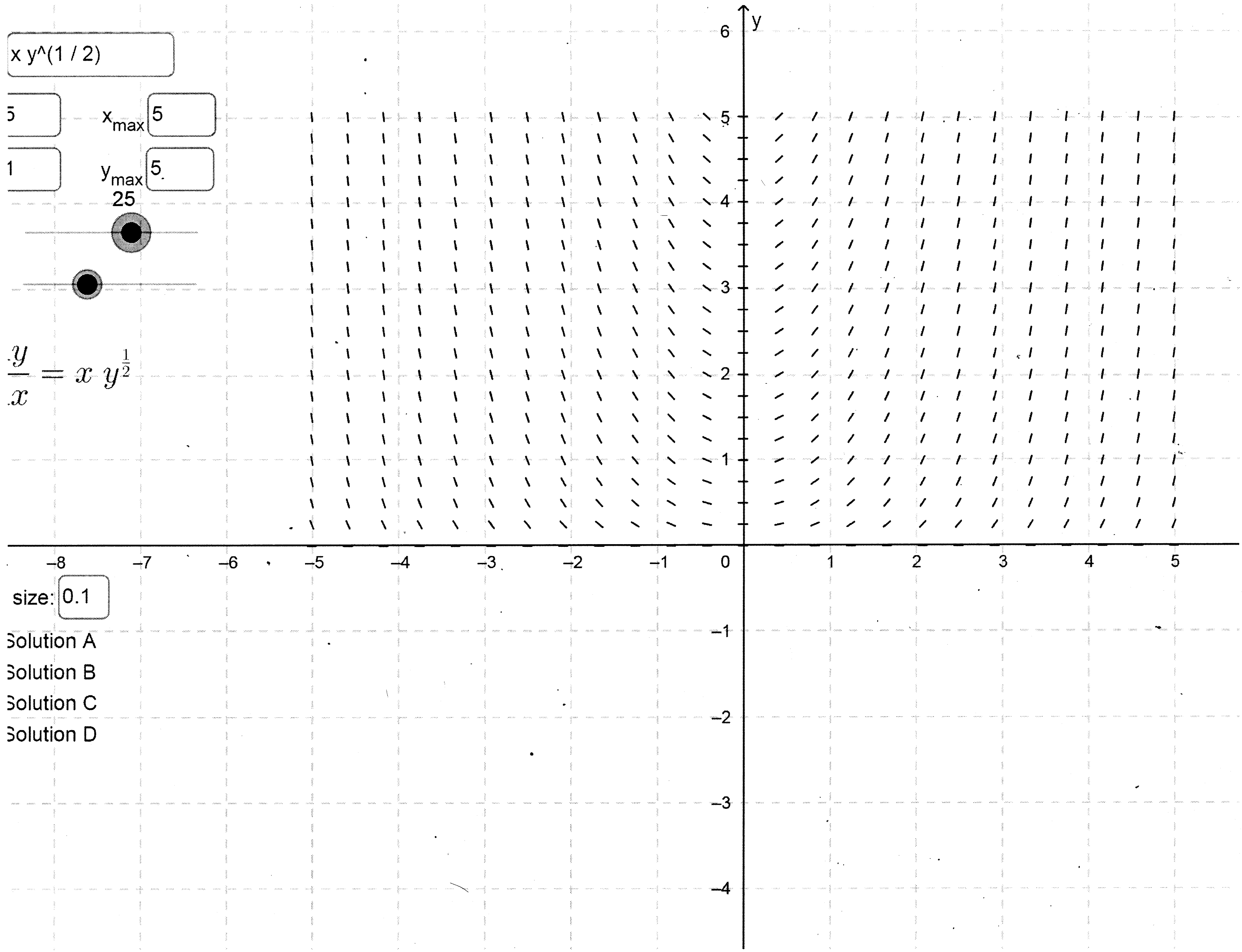
$$\mu(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = \frac{1}{x} \quad \text{ASSUMING } x > 0$$

$$\frac{1}{x}y = \int \left(\frac{1}{x^2} + \frac{1}{x}\right) dx = -\frac{1}{x} + \ln|x| + C$$

$$y = -1 + x\ln x + Cx, \quad x > 0$$

$$y(1) = 4 \Rightarrow -1 + 0 + C = 4 \Rightarrow C = 5$$

$$y(x) = 5x + x\ln x - 1$$


x y^(1 / 2)
5
x_{max} 5
1
y_{max} 5
25
$\frac{dy}{dx} = x\,y^{\frac{1}{2}}$
size: 0.1
Solution A
Solution B
Solution C
Solution D
y