Math 240 - Quiz 4

October 1, 2020

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due on October 6.

1. (3 points) Find the general solution: $y^{(4)} - 6y''' + 3y'' + 8y' + 48y = 0$
(Hint: $r^4 - 6r^3 + 3r^2 + 8r + 48 = (r^2 - 8r + 16)(r^2 + 2r + 3)$)

↑ CHAR EQN

$(r-4)^2 = 0$

$r = 4, 4$

$r^2 + 2r + 3 = 0$

$r = \frac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}\, i$

$$y(x) = c_1 e^{4x} + c_2 x e^{4x} + c_3 e^{-x} \cos\sqrt{2}x + c_4 e^{-x} \sin\sqrt{2}x$$

2. (3 points) Solve: $y'' - 6y' + 25y = 0, \quad y(0) = 3,\ y'(0) = 1$

CHAR EQN: $r^2 - 6r + 25 = 0$

$r^2 - 6r + 9 = -16$

$(r-3)^2 = -16$

$r = 3 \pm 4i$

$$y(x) = e^{3x}(3\cos 4x - 2\sin 4x)$$

$y(x) = c_1 e^{3x} \cos 4x + c_2 e^{3x} \sin 4x$

$y(0) = 3 \Rightarrow c_1 = 3$

$y'(x) = 9e^{3x}\cos 4x - 12e^{3x}\sin 4x + 3c_2 e^{3x}\sin 4x + 4c_2 e^{3x}\cos 4x$

$y'(0) = 1 \Rightarrow 9 + 4c_2 = 1 \Rightarrow c_2 = -2$

Turn over.

25 cm = 0.25 m

3. (4 points) A 20-kg mass is attached to a spring with spring constant 200 N/m. The damping constant for the system is 140 N-sec/m. The mass is pulled 25 cm to the right of equilibrium (stretching the spring) and given an initial leftward velocity of 1 m/sec. Solve for the equation of motion. When will the spring pass through equilibrium for the first time?

$$20x'' + 140x' + 200x = 0$$

$$x(0) = 0.25, \quad x'(0) = -1$$

$$x'' + 7x' + 10x = 0$$

$$r^2 + 7r + 10 = 0$$

$$(r+2)(r+5) = 0$$

$r = -2, \; r = -5$ System is overdamped.

$$x(t) = c_1 e^{-2t} + c_2 e^{-5t}$$

$$x(0) = 0.25 \Rightarrow c_1 + c_2 = 0.25$$

$$x'(t) = -2c_1 e^{-2t} - 5c_2 e^{-5t}$$

$$x'(0) = -1 \Rightarrow -2c_1 - 5c_2 = -1$$

$$5(c_1 + c_2 = 0.25)$$
$$-2c_1 - 5c_2 = -1$$

$$3c_1 = 0.25$$

$$c_1 = \frac{0.25}{3} = \frac{1}{12} \qquad c_2 = 0.25 - \frac{0.25}{3} = \frac{0.50}{3} = \frac{1}{6}$$

$$x(t) = \frac{1}{12} e^{-2t} + \frac{1}{6} e^{-5t}$$

$x(t)$ can never be zero!

The mass will never pass through equilibrium.