Math 240 - Quiz 7

October 29, 2020

Name key

Score

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due November 5.

1. (5 points) State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

$$(x^2-3)y''+2xy'=0$$

$$y'' + \frac{2x}{x^2-3}y' = 0 \Rightarrow \text{Singular pts at } -\sqrt{3} \text{ \& } \sqrt{3}$$

The power series solution is centered at $x=0$, so the radius of convergence is at least $\sqrt{3}$.

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y''(x) = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$(x^2-3)y''+2xy' = 0 =$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^n - \underbrace{\sum_{n=2}^{\infty} 3n(n-1)a_n x^{n-2}}_{\text{Replace } n \text{ by } n+2} + \sum_{n=1}^{\infty} 2na_n x^n$$

$$= \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=0}^{\infty} 3(n+2)(n+1)a_{n+2}x^n + \sum_{n=1}^{\infty} 2na_n x^n = 0$$

$$-6a_2 + (-18a_3 + 2a_1)x + \sum_{n=2}^{\infty}\left[(n(n-1)+2n)a_n - 3(n+2)(n+1)a_{n+2}\right]x^n$$

$$= 0$$

$$n(n-1)+2n = n(n+1)$$

a_0 and a_1 are arbitrary.

$$a_2 = 0, \quad a_3 = \frac{1}{9}a_1,$$

$$a_{n+2} = \frac{n}{3(n+2)}a_n, \quad n = 2,3,4,\ldots$$

Turn over.

2. (5 points) State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

$$5y'' - 2xy' + 10y = 0$$

$$\underbrace{y'' - \frac{2}{5}xy' + 2y = 0}$$

NO SINGULAR PTS.

$\boxed{\text{RADIUS OF CONVERGENCE OF OUR POWER SERIES SOLUTION IS } \infty.}$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y''(x) = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$5y'' - 2xy' + 10y = 0$$

$$= \underbrace{\sum_{n=2}^{\infty} 5n(n-1)a_n x^{n-2}}_{\text{REPLACE } n \text{ WITH } n+2} - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 10 a_n x^n$$

$$= \sum_{n=0}^{\infty} 5(n+2)(n+1)a_{n+2} x^n - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 10 a_n x^n$$

$$= 10a_2 + 10a_0 + \sum_{n=1}^{\infty} \left[5(n+2)(n+1)a_{n+2} + (10-2n)a_n \right] x^n$$

$$10a_2 + 10a_0 = 0$$

$$5(n+2)(n+1)a_{n+2} + (10-2n)a_n = 0$$

a_0 AND a_1 ARE ARBITRARY CONSTANTS.

$$a_2 = -a_0 \qquad a_{n+2} = \frac{2n-10}{5(n+2)(n+1)} a_n,$$

$$n = 1, 2, 3, \ldots$$