# Math 240 - Homework 1

September 9, 2021

Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 14.

1. (2 points) Solve the initial value problem.

$$\frac{dy}{dx} = 3x^2(y^2+1), \quad y(0) = 1$$

$$\frac{dy}{y^2+1} = 3x^2\,dx$$

$$\int \frac{1}{y^2+1}\,dy = \int 3x^2\,dx$$

$$\tan^{-1} y = x^3 + C$$

$x = 0,\ y = 1$

$$\tan^{-1} 1 = 0 + C$$

$$\frac{\pi}{4} = C$$

$$\tan^{-1} y = x^3 + \frac{\pi}{4}$$

$$y(x) = \tan\left(x^3 + \frac{\pi}{4}\right)$$

2. (2 points) A pitcher of buttermilk initially at 25°C is to be cooled by setting it on the front porch, where the temperature is 0°C. Suppose that the temperature of the buttermilk has dropped to 15°C after 20 min. When will it be at 5°C? (Use Newton's Law of Cooling.)

$$\frac{dT}{dt} = k(T - 0), \quad T(0) = 25$$

(0 = surrounding temp.)

$$\frac{dT}{dt} = kT, \quad T(0) = 25$$

Regular exponential decay...

$$T(t) = 25e^{kt}$$

$$T(20) = 15 = 25e^{20k}$$

$$k = \frac{\ln \frac{15}{25}}{20}$$

$$T(t) = 25e^{t \ln(\frac{3}{5})/20}$$

$$5 = 25e^{t \ln(\frac{3}{5})/20}$$

$$\ln\left(\frac{1}{5}\right) = t\,\frac{\ln\left(\frac{3}{5}\right)}{20}$$

$$t = \frac{20 \ln\left(\frac{1}{5}\right)}{\ln\left(\frac{3}{5}\right)}$$

Turn over.

$$\approx 63 \text{ min}$$

3. (3 points) Solve the differential equation: $xy' + (2x - 3)y = 4x^4$.

$$y' + \frac{2x-3}{x} y = 4x^3$$

$$\mu(x) = e^{\int (2 - \frac{3}{x})\,dx}$$

$$= e^{2x - 3\ln|x|} = e^{2x} \cdot \frac{1}{|x|^3}$$

$$\mu(x) = \frac{e^{2x}}{x^3} \quad \text{Assuming } x > 0$$

$$\mu(x)\, y(x) = \int \mu(x)\, q(x)\, dx$$

$$\frac{e^{2x}}{x^3} y = \int 4e^{2x}\, dx$$

$$= 2e^{2x} + C$$

$$y(x) = 2x^3 + Cx^3 e^{-2x}$$

4. (3 points) A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at the rate of 5 gal/s, and the well-mixed brine in the tank flows out at the rate of 3 gal/s. How much salt will the tank contain when it is full of brine? → FULL AFTER 150 SECONDS

$1\ \text{lb/gal} \cdot 5\ \text{gal/s} = 5\ \text{lb/s}$

$A(t)$ = AMOUNT OF SALT AT TIME $t$

$V(t)$ = VOLUME AT TIME $t$ = $100 + 2t$

$V(0) = 100$

$A(0) = 50$

$3\ \text{gal/s} \cdot \frac{A(t)}{V(t)}$

$$\frac{dA}{dt} = 5 - \frac{3A}{100+2t}$$

OR

$$\frac{dA}{dt} + \frac{3}{100+2t} A = 5$$

$$\mu(t) = e^{\int \frac{3}{100+2t}\,dt}$$

$$= e^{\frac{3}{2}\ln(100+2t)}$$

$$= (100+2t)^{3/2}$$

$$(100+2t)^{3/2} A(t) = \int 5(100+2t)^{3/2}\, dt$$

$$(100+2t)^{3/2} A(t) = (100+2t)^{5/2} + C$$

$$A(t) = (100+2t) + \frac{C}{(100+2t)^{3/2}}$$

$$A(0) = 50 \Rightarrow C = -50000$$

$$A(150) = 393.75 \text{ lb}$$