Math 240 - Quiz 1

August 26, 2021

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 31.

1. (3 points) Classify the differential equation by saying whether it is ordinary or partial, linear or nonlinear. Also give its order and name the dependent and independent variables. Finally, show that $y = \frac{1}{x} - \ln x$ is a solution.

$$x^2y'' + xy' - y = \ln x$$

2ND-ORDER, LINEAR, ORDINARY DE. INDEP. VARIABLE IS X.

DEP. VARIABLE IS y

VERIFY...

$y' = -\frac{1}{x^2} - \frac{1}{x}$

$y'' = \frac{2}{x^3} + \frac{1}{x^2}$

$$x^2y'' + xy' - y = x^2\left(\frac{2}{x^3} + \frac{1}{x^2}\right) + x\left(-\frac{1}{x^2} - \frac{1}{x}\right) - \left(\frac{1}{x} - \ln x\right)$$

$$= \frac{2}{x} + 1 - \frac{1}{x} - 1 - \frac{1}{x} + \ln x = \ln x \quad \checkmark$$

2. (3 points) Is the following ordinary differential equation linear or nonlinear? Explain how you know. Then verify that $y = \ln(x+C)$ is a solution for any constant C. Finally, determine the constant C so that $y(0) = 0$.

$$e^y y' = 1$$

THE EQUATION IS NONLINEAR. THE e^y MAKES IT NONLINEAR, AS WELL AS THE FACT THAT e^y IS MULTIPLIED BY y'.

VERIFY...

$$y' = \frac{1}{x+C} \Rightarrow e^y y' = e^{\ln(x+C)} \cdot \frac{1}{x+C} = (x+C)\left(\frac{1}{x+C}\right) = 1 \quad \checkmark$$

$$y(0) = 0 = \ln(0+C) = 0 \Rightarrow \boxed{C = 1}$$

3. (3 points) Solve the initial value problem: $\dfrac{dy}{dx} = xe^{-x}$, $y(0) = 1$.

$$y(x) = \int xe^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx$$

$u = x \quad du = dx$

$dv = e^{-x}dx \quad v = -e^{-x}$

$$= -xe^{-x} - e^{-x} + C$$

$$y(0) = 1 \Rightarrow 0 - e^{-0} + C = 1$$

$$\Rightarrow -1 + C = 1 \Rightarrow C = 2$$

$$\boxed{y(x) = -xe^{-x} - e^{-x} + 2}$$

4. (1 point) Write a differential equation that models the problem situation:

In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain disease is proportional to the product of the number who have the disease and the number who do not.

$$\boxed{\frac{dN}{dt} = kN(P-N)}$$