Math 240 - Quiz 2

September 2, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 7.

1. (4 points) Consider the differential equation $\frac{dy}{dx} = 3y^{2/3}$.

   (a) Referring to our existence/uniqueness theorem, explain why we should not expect this ODE to have a unique solution passing through any point where $y = 0$.

$f(x,y) = 3y^{2/3}$

$f_y(x,y) = 2y^{-1/3}$

f IS CONTINUOUS EVERYWHERE ON $\mathbb{R}^2$, BUT f_y IS NOT CONTINUOUS ALONG $y=0$. BASED ON OUR THEOREMS, WE EXPECT SOLUTIONS WHERE $y=0$, BUT NOT NECESSARILY A UNIQUE SOLUTION.

   (b) Use a slope field generator (see the links on our Lecture Resources page) to construct the slope field for the ODE.

SEE ATTACHED SHEET.

   (c) Find the solution(s) passing through $(0,0)$. Preferably, use your slope field to guess and check the solution(s) rather than use a solution technique.

ONE SOL'N

$y = x^3$

CHECK:

$y^{1/3} = x$

$\frac{1}{3}y^{-2/3}\frac{dy}{dx} = 1$ & $y(0) = 0$

$\frac{dy}{dx} = 3y^{2/3}$

2ND SOL'N

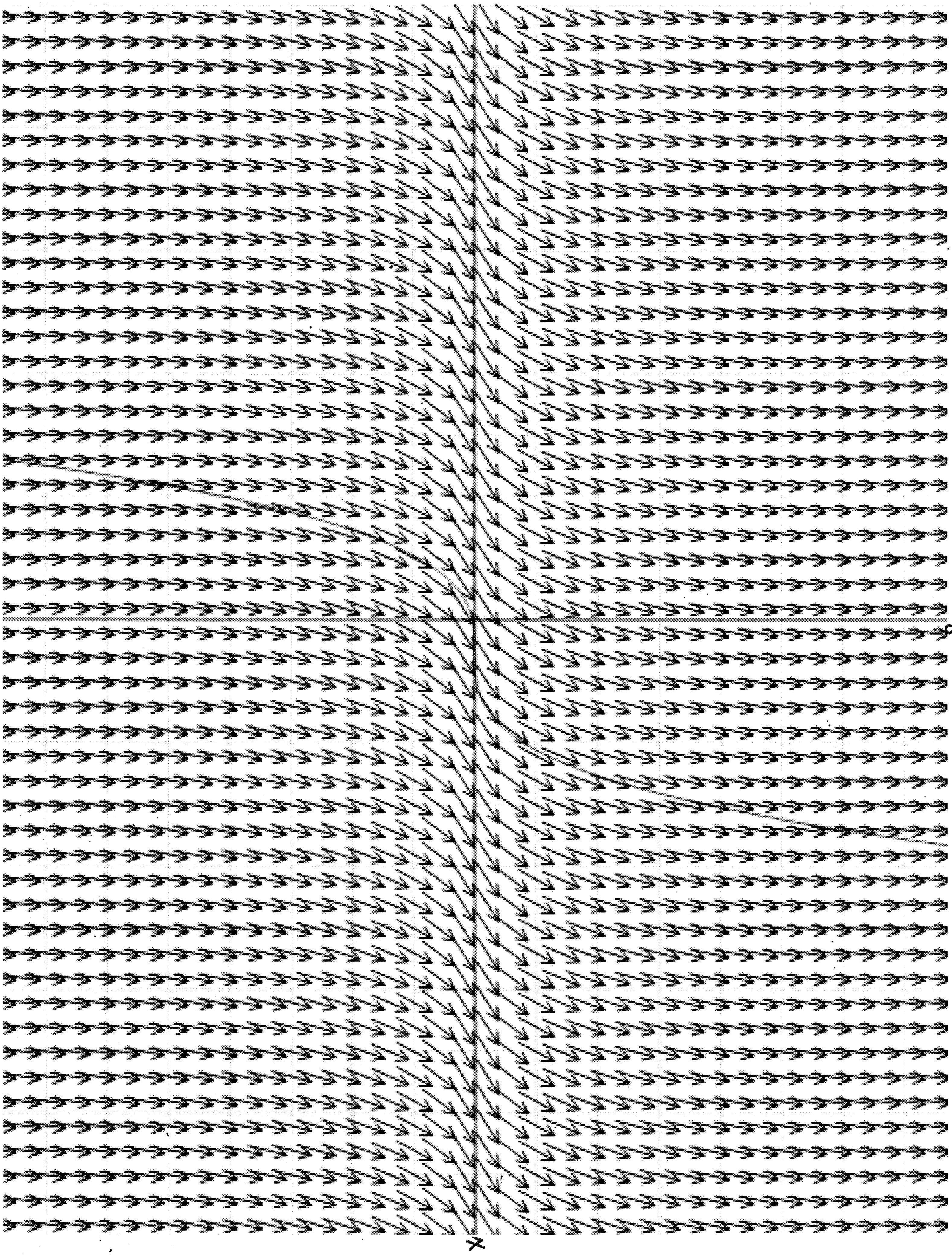
$y = 0$

CHECK:

$\left.\begin{array}{l}\frac{dy}{dx} = 0 \\ 3y^{2/3} = 0\end{array}\right\}$ SAME!

AND $y(0) = 0$

Turn over.



2. (3 points) Solve the initial value problem: $x\dfrac{dy}{dx} + y = y^2, \quad y(1) = \dfrac{1}{2}.$

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

$$\frac{1}{y(y-1)}\,dy = \frac{1}{x}\,dx$$

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$$

$$1 = A(y-1) + By$$

$$y = 0 \Rightarrow 1 = -1A \Rightarrow A = -1$$

$$y = 1 \Rightarrow 1 = B$$

$$\int \frac{1}{y-1} - \frac{1}{y}\,dy = \int \frac{1}{x}\,dx.$$

$$\ln|y-1| - \ln|y| = \ln|x| + C_1$$

$$\ln\left|\frac{y-1}{y}\right| = \ln|x| + C_1$$

$$\left|\frac{y-1}{y}\right| = C_2|x|$$

$$\frac{y-1}{y} = C_3 x$$

$$y(1) = \tfrac{1}{2} \Rightarrow C_3 = -1$$

$$y - 1 = -xy$$

$$y + xy = 1$$

$$\boxed{y = \frac{1}{x+1}}$$

3. (3 points) Solve the initial value problem: $\dfrac{dy}{dx} = \dfrac{1}{x + y^2}, \quad y(-2) = 0.$
(Hint: Write the differential equation for dx/dy and solve by letting y be the independent variable.)

$$\frac{dx}{dy} = x + y^2$$

$$\frac{dx}{dy} - x = y^2$$

$$\mu(y) = e^{\int -1\,dy} = e^{-y}$$

$$\mu(y)\,x(y) = \int \mu(y)\,y^2\,dy$$

$$x(y) = e^{y}\int y^2 e^{-y}\,dy$$

signs	u	dv
+	y^2	e^{-y}
−	$2y$	$-e^{-y}$
+	2	e^{-y}
−	0	$-e^{-y}$

$$x(y) = e^{y}\left[-y^2e^{-y} - 2ye^{-y} - 2e^{-y} + C\right]$$

$$x(y) = -y^2 - 2y - 2 + Ce^{y}$$

$x = -2$ when $y = 0$ $\Rightarrow$ $C = 0$

$$\boxed{x(y) = -y^2 - 2y - 2}$$