Math 240 - Quiz 3

September 23, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 28.

1. (2.5 points) Solve: $y^2y' + 2xy^3 = 6x$

Bernoulli. Let $u = y^3$

$$\frac{du}{dx} = 3y^2\frac{dy}{dx}$$

$$\frac{1}{3}\frac{du}{dx} + 2xu = 6x$$

$$\frac{du}{dx} + 6xu = 18x$$

$$\mu(x) = e^{\int 6x\,dx} = e^{3x^2}$$

$$e^{3x^2}u(x) = \int 18x e^{3x^2}\,dx$$

$u = 3x^2$

$du = 6x\,dx$

$$\int 3e^u\,du = 3e^u + C$$

$$= 3e^{3x^2} + C$$

$$u(x) = 3 + Ce^{-3x^2}$$

$$y(x) = \sqrt[3]{3 + Ce^{-3x^2}}$$

2. (2.5 points) Solve: $(x^2 - y^2)y' = 2xy$

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}\cdot\frac{\frac{1}{xy}}{\frac{1}{xy}}$$

$$\frac{dy}{dx} = \frac{2}{\frac{x}{y} - \frac{y}{x}}$$ Homogeneous.

Let $u = \frac{y}{x}$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{2}{\frac{1}{u} - u} = \frac{2u}{1-u^2}$$

$$x\frac{du}{dx} = \frac{2u}{1-u^2} - u$$

$$x\frac{du}{dx} = \frac{2u - u(1-u^2)}{1-u^2} = \frac{u^3 + u}{1-u^2}$$

$$\frac{1-u^2}{u^3+u}\,du = \frac{1}{x}\,dx$$

PFD... $\frac{1-u^2}{u^3+u} = \frac{A}{u} + \frac{Bu+C}{u^2+1}$

$$1 - u^2 = A(u^2+1) + (Bu+C)u$$

$A + B = -1$

$C = 0 \Rightarrow B = -2$

$A = 1$

Turn over.

$$\int\left(\frac{1}{u} - \frac{2u}{u^2+1}\right)du = \frac{1}{x}\,dx$$

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$$\ln|u| - \ln|u^2+1| = \ln|x| + C_1$$

$$\ln \frac{|u|}{u^2+1} = \ln|x| + C_1$$

$$\frac{|u|}{u^2+1} = |x| e^{C_1}$$

$$\frac{u}{u^2+1} = C_2 x$$

$$\frac{y/x}{y^2/x^2 + 1} = C_2 x$$

$$\frac{xy}{y^2 + x^2} = C_2 x$$

$$y = C_2 (x^2 + y^2)$$

LET'S LEAVE IT IMPLICIT.

3. (2.5 points) Solve: $y'' = (x + y')^2$

LET $u = y'$ SO THAT $u' = y''$.

$$u' = (x+u)^2$$

LET $\omega = x + u$

$$\omega' = 1 + u'$$

$$\omega' - 1 = \omega^2$$

$$\omega' = \omega^2 + 1$$

$$\frac{1}{\omega^2+1}\, d\omega = dx$$

$$\tan^{-1}\omega = x + C_1$$

$$x + u = \tan(x + C_1)$$

$$y' = \tan(x + C_1) - x$$

$$y(x) = -\ln|\cos(x+C_1)| - \frac{1}{2}x^2 + C_2$$

4. (2.5 points) Solve: $y'' = 2yy'$

LET $\frac{dy}{dx} = u$ SO THAT $\frac{d^2y}{dx^2} = \frac{du}{dx} = \frac{du}{dy}\cdot\frac{dy}{dx} = u\frac{du}{dy}$

$$u\frac{du}{dy} = 2yu$$

$$du = 2y\, dy$$

$$u = y^2$$

$$\frac{dy}{dx} = y^2 + C_1$$

$$\frac{dy}{y^2 + C_1} = dx$$

$$\frac{1}{\sqrt{C_1}}\tan^{-1}\frac{y}{\sqrt{C_1}} = x + C_2$$

$$\tan^{-1}\frac{y}{\sqrt{C_1}} = \sqrt{C_1}x + C_3$$

$$y(x) = \sqrt{C_1}\tan\left(\sqrt{C_1}x + C_3\right)$$

IF C_1 WOULD HAPPEN TO BE ZERO OR NEGATIVE, THIS WOULD REQUIRE DIFFERENT INTEGRATION.