Math 240 - Quiz 4

September 30, 2021

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 5.

1. (2.5 points) Given that $y(x) = xe^x$ is a particular solution of

$$y'' - y' - 12y = (1 - 12x)e^x,$$

find the general solution.

$$y'' - y' - 12y = 0$$

Char. eqn: $r^2 - r - 12 = 0$

$$(r-4)(r+3) = 0$$

$$r = 4, \quad r = -3$$

$$\{ e^{4x}, e^{-3x} \}$$

General solution

$$y(x) = C_1 e^{4x} + C_2 e^{-3x} + xe^x$$

2. (2.5 points) Find two linearly independent solutions of

$$y'' + 12y' + 36y = 0.$$

Compute the Wronskian to show that your solutions are independent.

Char. eqn: $r^2 + 12r + 36 = 0$

$$(r+6)(r+6) = 0$$

$$r = -6, \quad \text{mult } 2$$

$$y_1(x) = e^{-6x}$$

$$y_2(x) = xe^{-6x}$$

$$W[y_1, y_2](x) = \begin{vmatrix} e^{-6x} & xe^{-6x} \\ -6e^{-6x} & e^{-6x} - 6xe^{-6x} \end{vmatrix}$$

$$= e^{-12x} - 6xe^{-12x} + 6xe^{-12x}$$

$$= e^{-12x} \neq 0 \quad \text{For all } x.$$

Nonzero $W \Rightarrow$ Lin. Indep.

Turn over.

3. (2.5 points) Solve: $\dfrac{d^4y}{dx^4} - \dfrac{d^2y}{dx^2} = 0$

CHAR. EQN: $r^4 - r^2 = 0$

$$r^2(r^2 - 1) = 0$$

$$r^2(r+1)(r-1) = 0$$

$r = 0, \ r = 0, \ r = -1, \ r = 1$

$y_1 = 1$

$y_2 = x$

$y_3 = e^{-x}$

$y_4 = e^x$

$\boxed{y(x) = C_1 + C_2 x + C_3 e^{-x} + C_4 e^x}$

4. (2.5 points) Solve: $y'' - 4y' + 13y = 0; \quad y(0) = 1, \ y'(0) = 5$

CHAR. EQN: $r^2 - 4r + 13 = 0$

$$r^2 - 4r + 4 = -9$$

$$(r-2)^2 = -9$$

$$r - 2 = \pm 3i$$

$$r = 2 \pm 3i$$

$y_1 = e^{2x} \cos 3x$

$y_2 = e^{2x} \sin 3x$

$y(x) = C_1 e^{2x} \cos 3x + C_2 e^{2x} \sin 3x$

$y(0) = 1 \Rightarrow 1 = C_1$

$y'(x) = 2C_1 e^{2x} \cos 3x - 3C_1 e^{2x} \sin 3x$
$\qquad + 2C_2 e^{2x} \sin 3x + 3C_2 e^{2x} \cos 3x$

$y'(0) = 5 \Rightarrow 5 = 2C_1 + 3C_2$

$\qquad\qquad\qquad = 2 + 3C_2 \Rightarrow C_2 = 1$

$\boxed{y(x) = e^{2x} \cos 3x + e^{2x} \sin 3x}$