Math 240 - Quiz 5

October 21, 2021

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 26.

1. (5 points) Solve: $y'' - y' - 6y = 4\cos 3x$, $y(0) = 1, y'(0) = -1$

Homo. eqn:

$y'' - y' - 6y = 0$

$r^2 - r - 6 = 0$

$(r-3)(r+2) = 0$

$y_h(x) = c_1 e^{3x} + c_2 e^{-2x}$

Non Homo. eqn:

$g(x) = 4\cos 3x$

$y_p(x) = x^s (A\cos 3x + B\sin 3x)$

$s = 0$

$y_p(x) = A\cos 3x + B\sin 3x$

$y_p'(x) = -3A\sin 3x + 3B\cos 3x$

$y_p''(x) = -9A\cos 3x - 9B\sin 3x$

$y'' - y' - 6y = 4\cos 3x$

$\Downarrow$

$-9A\cos 3x - 9B\sin 3x + 3A\sin 3x$

$\quad - 3B\cos 3x - 6A\cos 3x - 6B\sin 3x$

$\quad\quad = 4\cos 3x$

$-15A - 3B = 4$

$-15B + 3A = 0 \Rightarrow A = 5B$

$-15(5B) - 3B = 4$

$-78B = 4$

$B = -\frac{4}{78} = -\frac{2}{39}$

$A = -\frac{10}{39}$

$y(x) = c_1 e^{3x} + c_2 e^{-2x} - \frac{10}{39}\cos 3x$

$\quad\quad - \frac{2}{39}\sin 3x$

$y(0) = 1 \Rightarrow c_1 + c_2 - \frac{10}{39} = 1$

$y'(0) = -1 \Rightarrow 3c_1 - 2c_2 - \frac{6}{39} = -1$

$c_1 + c_2 = \frac{49}{39}$ $c_2 = \frac{49}{39} - \frac{1}{3}$

$3c_1 - 2c_2 = -\frac{33}{39}$ $= \frac{36}{39} = \frac{12}{13}$

$\overline{}$

$5c_1 = \frac{65}{39}$ *Turn over.*

$c_1 = \frac{1}{3}$

$y(x) = \frac{1}{3}e^{3x} + \frac{12}{13}e^{-2x} + \frac{10}{39}\cos 3x + \frac{2}{39}\sin 3x$

2. (2 points) Consider the following equation:

$$y'' - 12y' + 36y = (3x+5)e^{6x}.$$

Solve the corresponding homogeneous equation. Then use your undetermined coefficients table to find the appropriate <u>form</u> of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

$$y'' - 12y' + 36y = 0$$

$$r^2 - 12r + 36 = 0$$

$$(r-6)^2 = 0$$

$$y_h(x) = C_1 e^{6x} + C_2 x e^{6x}$$

$$g(x) = (3x+5)e^{6x}$$

$$y_p(x) = x^s(Ax+B)e^{6x}.$$

MUST CHOOSE $S = 2$

$$\boxed{y_p(x) = (Ax^3 + Bx^2)e^{6x}}$$

3. (3 points) Use variation of parameters to solve $y'' + 9y = \csc 3x$.

Homo. Eqn: $y'' + 9y = 0$

$$r^2 + 9 = 0$$

$$r = \pm 3i$$

$$y_h(x) = C_1 \cos 3x + C_2 \sin 3x$$

Non Homo. Eqn :

$$g(x) = \csc 3x$$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}$$

$$= 3$$

$$V_1(x) = \int \frac{-\csc 3x \, \sin 3x}{3} dx = \int -\frac{1}{3} dx$$

$$= -\frac{1}{3}x$$

$$V_2(x) = \int \frac{\csc 3x \, \cos 3x}{3} dx$$

$$= \int \frac{1}{3}\cot 3x \, dx$$

$$= \frac{1}{9}\ln|\sin 3x|$$

$$y_p(x) = -\frac{1}{3}x\cos 3x + \frac{1}{9}\ln|\sin 3x| \cdot \sin 3x$$

$$\boxed{y(x) = C_1\cos 3x + C_2\sin 3x - \frac{1}{3}x\cos 3x + \frac{1}{9}\ln|\sin 3x|\sin 3x}$$