Math 240 - Test 2

October 14, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand, unless otherwise specified.

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1. (12 points) Solve: $\dfrac{dy}{dx} = \dfrac{y^4 - x^4}{2xy^3}$

2. (12 points) Solve: $\dfrac{dy}{dx} - y = e^x y^2$

3. (15 points) Consider the equation $xy'' + 5y' = 0,\ x > 0$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = \dfrac{1}{x^4}$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Now consider the nonhomogeneous equation $xy'' + 5y' = 12x,\ x > 0$. Verify that $y_p(x) = x^2$ is a solution.

(d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $xy'' + 5y' = 12x;\ y(1) = 2,\ y'(1) = 10$.

(e) Is your solution in part (d) unique? Explain.

4. (8 points) Solve the following initial value problem.

$$y'' + y' - 12y = 0; \quad y(0) = 3,\ y'(0) = 23$$

5. (8 points) Find the general solution: $\quad y^{(5)} + 2y^{(3)} = 0$

6. (8 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

   (a) $x(t) = 6e^{-t/3}\sin(\sqrt{5}t + \frac{1}{2})$

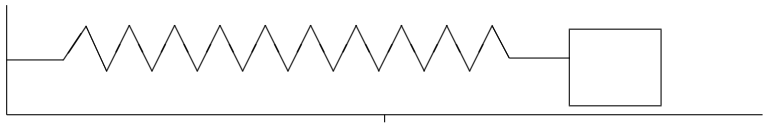
   (b) $x(t) = 7e^{-3t} - 9te^{-3t}$

   (c) $5x'' + 8x' + 2x = 0$

   (d) $x'' + 8x = 0$

7. (5 points) Let $y_1(x) = x + 1$ and $y_2(x) = x^2 - (x + 2)^2$. Compute the Wronskian of y_1 and y_2. Briefly explain why $y(x) = c_1y_1(x) + c_2y_2(x)$ cannot be the general solution of a 2nd-order, linear, homogeneous differential equation.

8. (12 points) A 1-kg mass is attached to a spring with spring constant $\frac{5}{4}$ N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the right at $\frac{1}{2}$ m/sec. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



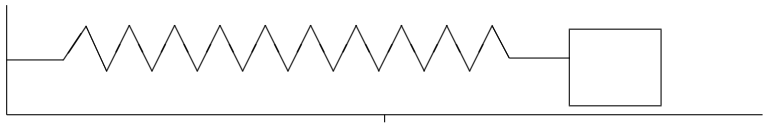
$x = 0$
(Equilibrium)

The following problems make up the take-home portion of the test. These problems are due October 19, 2021. You must work on your own.

9. (8 points) Solve the following initial value problem.

$$y'' = y' \cdot e^y; \quad y(0) = 0,\ y'(0) = 1$$

10. (12 points) A 9-kg mass is attached to a spring with spring constant 37 N/m. The damping constant for the system is 6 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the left at 2 m/sec. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift. When does the mass pass through equilibrium for the second time?



$x = 0$
(Equilibrium)