Math 240 - Test 3

November 11, 2021

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations where necessary. All integration must be done by hand, unless otherwise specified. You must work individually on this test. The test is due November 16.

1. (16 points) Use undetermined coefficients to solve the following equation.

$$2y'' + 6y' - 20y = 60 \sin 2x$$

2. (16 points) Use variation of parameters to solve the following equation.

$$y'' + 3y' + 2y = \sin(e^x)$$

3. (8 points) Consider the following equation:

$$y'' - 10y' + 25y = 5x^2e^{5x}.$$

Solve the corresponding homogeneous equation. Then use your table to find the appropriate form of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

4. (8 points) Use the definition of the Laplace transform to find the transform of $f(t) = te^{-t}$. Show all work.

5. (16 points) State the recurrence relation that describes the coefficients of the power series solution (centered at $x = 0$), and state the guaranteed (by our theorem) radius of convergence.

$$(x^2 - 1)y'' + 2xy' + 2xy = 0$$

6. (16 points) Find a power series solution centered at $x = 0$. Then refer to the Common Infinite Series sheet (available on the Lecture Resources page) to find a familiar expression for your solution and its interval of convergence.

$$(x-2)y' + y = 0$$

7. (6 points) Use your table of Laplace transforms to determine the transform of each function. Show how you got your answer.

   (a) $f(t) = 2 + 5e^{2t} - \cos 4t$

   (b) $f(t) = t^{3/2} - t \sin 2t$

8. (9 points) Find the inverse Laplace transform of each function. Show how you got your answer.

   (a) $F(s) = \dfrac{3s+1}{s^2+4}$

   (b) $F(s) = 5s^{-1}e^{-3s}$

   (c) $F(s) = \dfrac{3}{s} - \dfrac{2}{s^4} - \dfrac{8}{6-s}$

9. (5 points) Consider the differential equation $x^2y'' - 3xy' + 8y = 0$. Explain why it might not be wise to look for a power series solution centered at $x = 0$. Then say how you would solve the equation (but do not solve it).