

Math 240 - Final Exam A

December 9, 2021

Name _____

Score _____

Show all work to receive full credit. You must work individually. This test is due December 14. If your approach to any problem on the test requires a partial fraction decomposition, you may use technology to find your PFD. All integration must be done by hand, unless otherwise indicated.

1. (10 points) A tank initially contains 50 gal of pure water. A salt solution containing 2 lb of salt per gallon enters the tank at 3 gal/min and is uniformly mixed. The mixed solution leaves the tank at 2 gal/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up and solve the appropriate initial value problem to determine $A(t)$. Use technology to sketch the graph of $A(t)$ for $0 \leq t \leq 30$. Attach your graph.

2. (16 points) The following differential equation falls into at least three of the named types of equations in section 1.6. Use two different approaches from section 1.6 to solve the equation. Use a word or short phrase to describe each approach.

$$3xy^2 \frac{dy}{dx} + y^3 = x^3$$

(a) First approach:

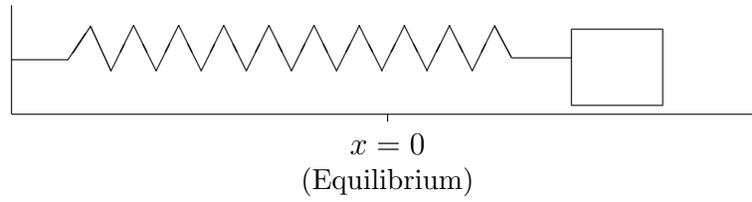
(b) Second approach:

3. (14 points) Consider the equation $y' + y = e^x$. This semester, we have studied at least four different ways to solve this equation. Solve the equation using two different approaches. Use a word or short phrase to describe each approach.

(a) First approach:

(b) Second approach:

4. (10 points) A 1-kg mass is attached to a spring with spring constant 3 N/m. The damping constant for the system is 2 N-sec/m. The mass is moved 0.5 m to the left of equilibrium (compressing the spring) and pushed to the right at 1 m/s. Set up and solve the initial value problem that describes the motion. Write your final answer in terms of a single trigonometric function with phase shift.



5. (10 points) Solve the following one-dimensional heat equation with Neumann boundary conditions. Rather than derive the solution method (as we did in class), you can use Theorem 2 on page 596. (You may use technology to evaluate the required integrals.)

$$5 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 10, \quad t > 0,$$

$$u_x(0, t) = u_x(10, t) = 0,$$

$$u(x, 0) = 4x$$

Math 240 - Final Exam B

December 16, 2021

Name _____

Score _____

Show all work to receive full credit. All integration must be done by hand.

1. (10 points) Choose either ONE of following equations (A or B). Circle your choice and solve it. Ignore the other equation.

(A) $(e^{2y} - y \cos xy) dx + (2xe^{2y} - x \cos xy + 2y) dy = 0$

(B) $x^2 y' + x(x + 2)y = e^x$

2. (10 points) Solve the differential equation.

$$y'' + 2y' = 2x + 5 - e^{-x}$$

3. (10 points) State the recurrence relation that describes the coefficients of the power series solution centered at $x = 0$.

$$y'' - 2xy = 0$$

4. (10 points) Use Laplace transform techniques to solve the initial value problem.

$$y'' - 4y' + 4y = t^3 e^{2t}; \quad y(0) = 0, \quad y'(0) = 0$$