

Math 240 - Quiz 10

November 16, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Use Laplace transforms to solve the IVP. You may use a computer algebra system for any partial fraction decompositions.

$$y'' + 4y' + 6y = 1 + e^{-t}, \quad y(0) = 0, \quad y'(0) = 0$$

$$s^2 Y + 4sY + 6Y = \frac{1}{s} + \frac{1}{s+1}$$

$$(s^2 + 4s + 6)Y = \frac{1}{s} + \frac{1}{s+1}$$

$$Y(s) = \frac{\frac{1}{s} + \frac{1}{s+1}}{s^2 + 4s + 6} = -\frac{1}{6} \left(\frac{3s+10}{(s+2)^2 + 2} \right) + \frac{1}{3} \frac{1}{s+1} + \frac{1}{6} \frac{1}{s}$$

$$= -\frac{1}{6} \left(\frac{3s+6}{(s+2)^2 + 2} + \frac{4}{(s+2)^2 + 2} \right) + \frac{1}{3} \frac{1}{s+1} + \frac{1}{6} \frac{1}{s} = -\frac{1}{2} \frac{s+2}{(s+2)^2 + 2} - \frac{2}{3\sqrt{2}} \frac{\sqrt{2}}{(s+2)^2 + 2}$$

$$y(t) = -\frac{1}{2} e^{-2t} \cos \sqrt{2}t - \frac{\sqrt{2}}{3} e^{-2t} \sin \sqrt{2}t + \frac{1}{3} e^{-t} + \frac{1}{6}$$

2. (5 points) Use Laplace transforms to solve the system of equations.

$$x' = -x + y, \quad x(0) = 0$$

$$y' = 2x, \quad y(0) = 1$$

$$sX = -X + Y$$

$$sY - 1 = 2X$$

$$(s+1)X - Y = 0$$

$$-2X + sY = 1$$

$$(s^2 + s)X - 2X = 1$$

$$X = \frac{1}{s^2 + s - 2}$$

$$Y = sX + X = \frac{s}{s^2 + s - 2} + \frac{1}{s^2 + s - 2}$$

$$X = \frac{1}{(s+2)(s-1)} = \frac{A}{s+2} + \frac{B}{s-1}$$

$$Y = \frac{s+1}{(s+2)(s-1)} = \frac{C}{s+2} + \frac{D}{s-1}$$

Cover-up...

$$A = -\frac{1}{3}$$

$$B = \frac{1}{3}$$

$$C = \frac{1}{3}$$

$$D = \frac{2}{3}$$

$$x(t) = -\frac{1}{3} e^{-2t} + \frac{1}{3} e^t$$

$$y(t) = \frac{1}{3} e^{-2t} + \frac{2}{3} e^t$$