Math 240 - Quiz 11

November 30, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) **Use convolution** to find the inverse transform of $Y(s) = \dfrac{14}{(s+2)(s-5)}$.

$$Y(s) = \frac{14}{s+2} \cdot \frac{1}{s-5}$$

$$y(t) = 14 \int_0^t e^{-2\tau} e^{5(t-\tau)} \, d\tau$$

$$= 14 \int_0^t e^{5t} e^{-7\tau} \, d\tau$$

$$14 e^{5t} \int_0^t e^{-7\tau} \, d\tau$$

$$= 14 e^{5t} \left(-\frac{1}{7} e^{-7\tau} \right) \Big|_{\tau=0}^{\tau=t}$$

$$= 14 e^{5t} \left(-\frac{1}{7} e^{-7t} + \frac{1}{7} \right)$$

$$= \boxed{2e^{5t} - 2e^{-2t}}$$

2. (2 points) Use the derivative-of-transform theorem to compute the Laplace transform of $f(t) = te^{-t}\sin 2t$. Use your table to check that your answer is correct.

$$\mathcal{L}\{f(t)\}(s) = (-1) \frac{d}{ds} \left(\frac{2}{(s+1)^2 + 4} \right) = \frac{4(s+1)}{[(s+1)^2 + 4]^2}$$

$$\mathcal{L}\{te^{-t} \sin 2t\}(s) = \frac{4(s+1)}{[(s+1)^2 + 4]^2}$$

3. (6 points) Use Laplace transforms to solve. (Use the back if necessary.)

$$y'' + 3ty' - 6y = 1; \quad y(0) = 0, \ y'(0) = 0$$

$$s^2 Y + 3(-1) \frac{d}{ds} (sY) - 6Y = \frac{1}{s}$$

$$s^2 Y - 3sY' - 3Y - 6Y = \frac{1}{s}$$

$$-3sY' + (s^2 - 9)Y = \frac{1}{s}$$

$$Y' - \frac{s^2 - 9}{3s} Y = -\frac{1}{3s^2}$$

$$\mu(s) = e^{\int \left(\frac{3}{s} - \frac{s}{3}\right) ds}$$

$$= e^{3\ln s - \frac{1}{6}s^2} = s^3 e^{-\frac{1}{6}s^2}$$

$$\mu(s) Y(s) = \int \mu(s) Q(s) \, ds$$

$$s^3 e^{-\frac{1}{6}s^2} Y(s) = \int -\frac{s}{3} e^{-\frac{1}{6}s^2} \, ds$$

$$= e^{-\frac{1}{6}s^2} + C$$

$$Y(s) = \frac{1}{s^3} + \frac{Ce^{\frac{1}{6}s^2}}{s^3}$$

MUST HAVE $C = 0$ FOR $\lim_{s \to \infty} Y(s) = 0$

$$Y(s) = \frac{1}{s^3} \Rightarrow \boxed{y(t) = \frac{1}{2}t^2}$$