

Math 240 - Quiz 12

December 7, 2023

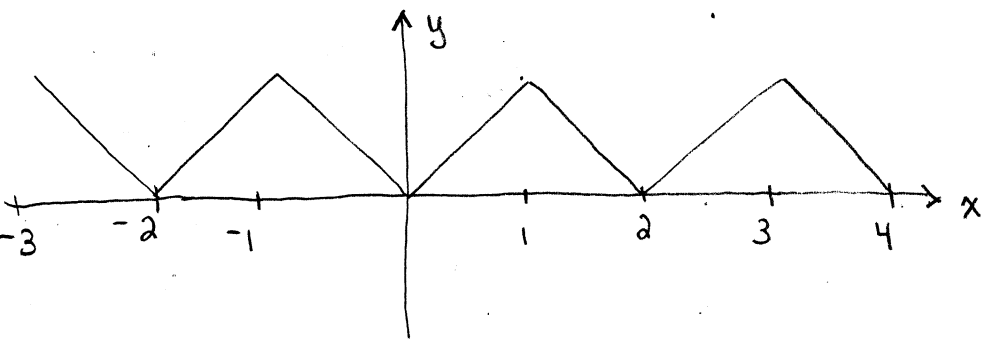
Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Let $f(x)$ be the periodic extension (with period 2) of its portion defined on $[-1, 1]$ by $f(x) = |x|$.

(a) Roughly sketch the graph of 3 or 4 periods of f .



(b) Determine the Fourier series for f .

We are essentially finding the Fourier cosine series for

$$g(x) = x, \quad 0 \leq x \leq 1.$$

$$a_0 = 2 \int_0^1 x \, dx = 2 \left(\frac{1}{2} \right) = 1$$

$$\begin{aligned} a_n &= 2 \int_0^1 x \cos(n\pi x) \, dx = 2 \left(\frac{x}{n\pi} \sin(n\pi x) + \frac{1}{n^2 \pi^2} \cos(n\pi x) \right) \Big|_0^1 \\ &= 2 \left(0 + \frac{\cos(n\pi)}{n^2 \pi^2} - 0 - \frac{1}{n^2 \pi^2} \right) \\ &= \frac{2(-1)^n - 2}{n^2 \pi^2} ; \quad n=1, 2, 3, \dots \end{aligned}$$

$$f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2(-1)^n - 2}{n^2 \pi^2} \cos(n\pi x)$$

The graph of the Fourier series through $n=100$ is attached.

