Math 240 - Quiz 1

August 24, 2023

Name key

Score __________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 29.

1. (4 points) State whether each equation is ordinary or partial, linear or nonlinear, give its order, and say which variable is dependent.

   (a) $\dfrac{dp}{dt} = kp(P_0 - p) = kP_0p - kp^2$

   ORDINARY, NONLINEAR, 1^{ST} ORDER, DEP. VAR. IS p

   (b) $8\,\dfrac{d^4y}{dx^4} = x(1-x)$

   ORDINARY, LINEAR, 4^{TH} ORDER, DEP. VAR. IS y

   (c) $\dfrac{\partial N}{\partial t} = \dfrac{\partial^2 N}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial N}{\partial r} + kN$

   PARTIAL, LINEAR, 2^{ND} ORDER, DEP. VAR. IS N

   (d) $\sqrt{1-x}\,\dfrac{d^2x}{dt^2} + 2t\dfrac{dx}{dt} = 0$

   ORDINARY, NONLINEAR, 2^{ND} ORDER, DEP. VAR. IS x

2. (2 points) Show that $x^2y + y^2 = c$ is an implicit solution of $2xy\,dx + (x^2 + 2y)\,dy = 0$.

$$\frac{d}{dx}(x^2y + y^2) = 0$$

$$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2xy\,dx + x^2\,dy + 2y\,dy = 0$$

$$2xy\,dx + (x^2 + 2y)\,dy = 0$$

IT CHECKS OUT!

Turn over.

3. (3 points) Solve the initial value problem: $\frac{dy}{dx} = \frac{1}{1-x^2}$, $y(3) = \ln 8$

Could simply say $y(x) = \tanh^{-1} x + C$,

but I'll use a PFD.

$$\frac{1}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$$

Cover up $\Rightarrow$ $A = \frac{1}{2}$, $B = \frac{1}{2}$

$$y(x) = \int \frac{1/2}{1-x} + \frac{1/2}{1+x}\, dx$$

$$= -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + C$$

$y(3) = \ln 8$

$$\Rightarrow \ln 8 = -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 4 + C$$

$$3\ln 2 = -\frac{1}{2}\ln 2 + \ln 2 + C$$

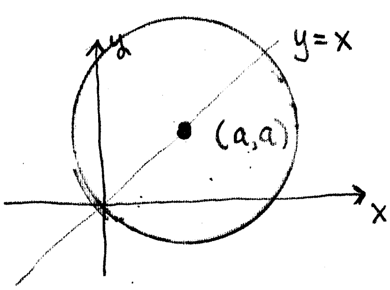
$$C = \frac{5}{2}\ln 2$$

$$y(x) = \frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x| + \frac{5}{2}\ln 2$$

4. (1 point) This problem may be challenging so it's only worth one point.

Find a differential equation for the family of circles passing through the origin with centers on the line $y = x$.

Your final answer should contain only x's and y's, no other parameters or constants.



$$(x-a)^2 + (y-a)^2 = 2a^2$$

$$\Downarrow$$

$$2(x-a) + 2(y-a)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x-a}{y-a}$$

Now let's solve for a...

$$x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = 2a^2$$

$$x^2 + y^2 = 2ax + 2ay$$

$$a = \frac{x^2+y^2}{2x+2y}$$

$$\frac{dy}{dx} = -\frac{x - \frac{x^2+y^2}{2x+2y}}{y - \frac{x^2+y^2}{2x+2y}}$$

Could clean this up, but let's not.