Math 240 - Quiz 2

Name key

August 31, 2023

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 5.

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1. (5 points) Consider the equation $\frac{dy}{dx} = 2xy^2 + 3x^2y^2$.

   (a) Use our existence/uniqueness theorem to say what you can about possible solutions through a given point.

$f(x,y) = 2xy^2 + 3x^2y^2$

$f_y(x,y) = 4xy + 6x^2y$

f & f_y ARE CONTINUOUS EVERYWHERE.

THERE IS A UNIQUE SOLUTION THROUGH ANY GIVEN POINT.

   (b) Use a slope field generator to construct a slope field for the equation in the vicinity of the point $(1, -1)$. Print and attach your slope field (or email it).

SEE ATTACHED.

   (c) Solve the equation along with the initial condition $y(1) = -1$.

$$\frac{1}{y^2}\,dy = (2x + 3x^2)\,dx$$

$$\int y^{-2}\,dy = \int 2x + 3x^2\,dx$$

$$-\frac{1}{y} = x^2 + x^3 + C_1$$

$$y = \frac{1}{C_2 - x^2 - x^3}$$

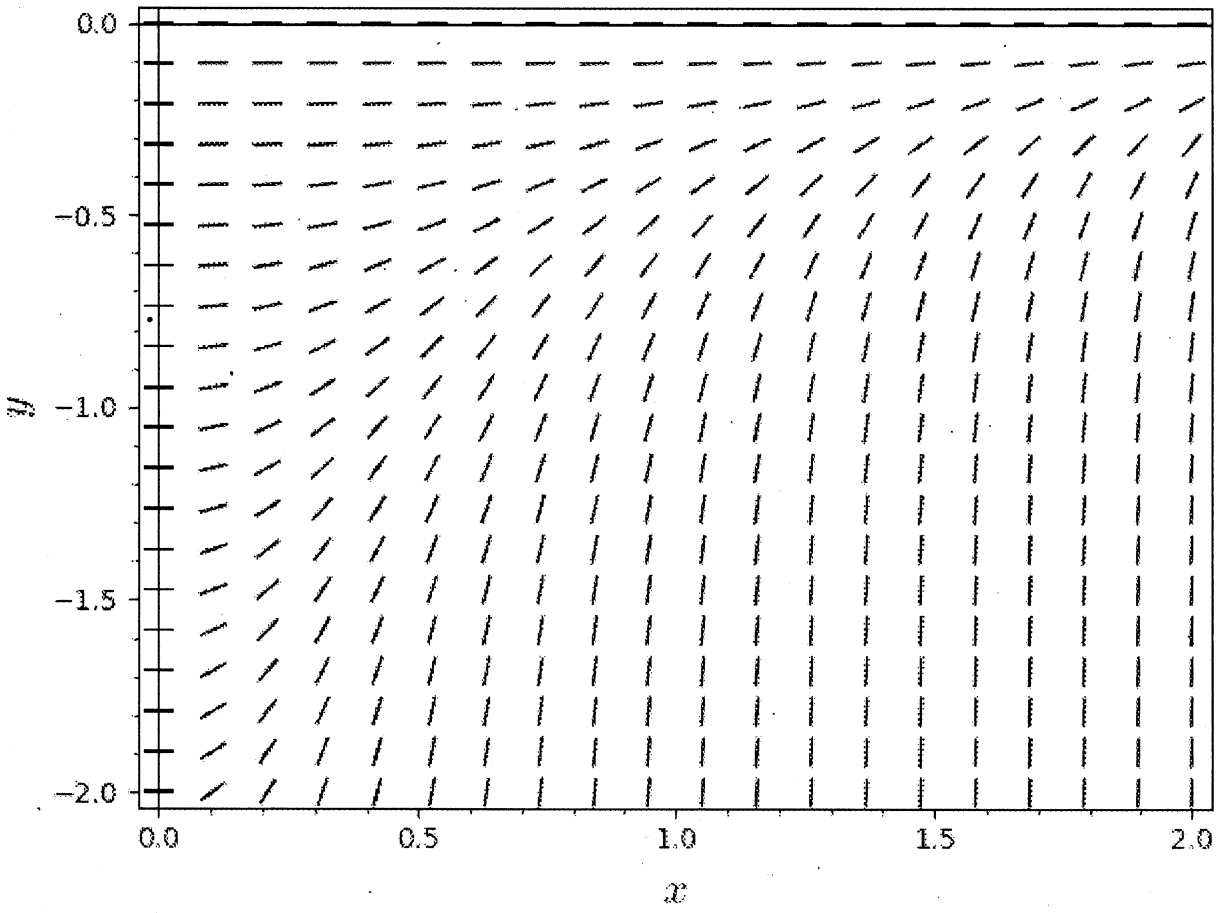
$$y(x) = \frac{1}{1 - x^2 - x^3}$$

$y(1) = -1 \Rightarrow \frac{1}{C_2 - 2} = -1$

$\Rightarrow C_2 = 1$

Turn over.

Slope Field For Problem #1b



2. (1 point) Use Euler's method (preferably on a calculator or computer) with $h = 0.1$ to estimate $y(2)$ for the IVP

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2, \quad y(1) = -1.$$

See attached.

3. (2 points) Use Euler's method (by hand) with $h = 0.1$ to estimate $y(0.3)$ for the IVP

$$y' = -2xy, \quad y(0) = 2.$$

$f(x,y) = -2xy$

$y_0 = 2$

$x_0 = 0$

$y_1 = 2 + 0.1\left[(-2)(0)(2)\right] = 2$

$x_1 = 0.1$

$y_2 = 2 + 0.1\left[-2(0.1)(2)\right] = 1.96$

$x_2 = 0.2$

$y_3 = 1.96 + 0.1\left[-2(0.2)(1.96)\right]$

$= 1.8816$

$x_3 = 0.3$

$y(0.3) \approx 1.8816$

4. (2 points) Find the exact solution of the IVP in problem 3.

$$\frac{dy}{y} = -2x\,dx$$

$\ln|y| = -x^2 + C_1$

$|y| = e^{-x^2 + C_1} = C_2 e^{-x^2}$

$y = C_3 e^{-x^2}$

$y(0) = 2 \Rightarrow C_3 = 2$

$y(x) = 2e^{-x^2}$

$y(0.3) = 2e^{-0.09} \approx 1.82786$

Problem #2

Starting values:
x = 1.0 , y = -1.0

Ending values:
x = 2.000000000000001 , y = -0.07782801221302064

[1.0, -1.0]
[1.1, -0.5]
[1.2000000000000002, -0.35424999999999995]
[1.3000000000000003, -0.26991866199999999]
[1.4000000000000004, -0.21403804549846744]
[1.5000000000000004, -0.17427298218720955]
[1.6000000000000005, -0.14466118667479666]
[1.7000000000000006, -0.12189276415878217]
[1.8000000000000007, -0.10395934409198009]
[1.9000000000000008, -0.08956369385356583]
[2.000000000000001, -0.07782801221302064]

Graph to accompany Problem #2