Math 240 - Quiz 5

September 28, 2023

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Consider the equation $yy'' = 6x^4$. Show that $y(x) = x^3$ is a solution, and show that $y(x) = 2x^3$ is NOT a solution. Why is a linear combination of solutions NOT a solution?

$y = x^3$, $y' = 3x^2$, $y'' = 6x$, $yy'' = 6x^4$ ✓. $y = 2x^3$, $y' = 6x^2$, $y'' = 12x$, $yy'' = 24x^4 \neq 6x^4$ ✓

THE EQUATION IS NOT LINEAR AND NOT HOMOGENEOUS.

2. (3 points) Suppose a and b are real numbers with $a \neq b$. Compute the Wronskian of $y_1(x) = e^{ax}$ and $y_2(x) = e^{bx}$.

$$W[y_1, y_2](x) = \begin{vmatrix} e^{ax} & e^{bx} \\ ae^{ax} & be^{bx} \end{vmatrix} = b e^{(a+b)x} - a e^{(a+b)x} = \boxed{(b-a) e^{(a+b)x}}$$

(NOTICE THAT W IS NEVER ZERO)

3. (2 points) It is easy to verify (don't bother) that $y_1(x) = 1$ and $y_2(x) = e^x$ are solutions of $y'' - y' = 0$. Find another solution.

LINEAR, HOMOGENEOUS

$y(x) = c_1 + c_2 e^x$ IS A SOLUTION FOR ANY c_1 & c_2

e.g., $\boxed{y(x) = \pi + \sqrt{2}\, e^x}$

4. (2 points) It is easy to verify (don't bother) that $y_1(x) = x^2$ and $y_2(x) = x^3$ are two **different**, linearly independent solutions of the initial value problem

$$x^2 y'' - 4xy' + 6y = 0; \quad y(0) = 0,\ y'(0) = 0.$$

Explain why does this not contradict our existence/uniqueness theorem for linear equations?

OUR EXISTENCE/UNIQUENESS THEOREM REQUIRES THE LEADING COEFFICIENT FUNCTION TO BE THE CONSTANT FUNCTION $a(x) \equiv 1$.