Math 240 - Quiz 6

October 5, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the general solution: $y''' + 6y'' + 9y' = 0$

Char eqn: $r^3 + 6r^2 + 9r = 0$

$r(r+3)(r+3) = 0$

$r = 0, \; r = -3$ (twice)

$y_1 = e^{0x}, \; y_2 = e^{-3x}, \; y_3 = xe^{-3x}$

$y(x) = c_1 + c_2 e^{-3x} + c_3 x e^{-3x}$

2. (3 points) Find the general solution: $y'' - 4y' + 13y = 0$

Char. eqn: $r^2 - 4r + 13 = 0$

$r^2 - 4r + 4 = -9$

$(r-2)^2 = -9$

$r = 2 \pm 3i$

$\alpha = 2, \; \beta = 3$

$y_1 = e^{2x}\cos 3x$

$y_2 = e^{2x}\sin 3x$

$y(x) = c_1 e^{2x}\cos 3x + c_2 e^{2x}\sin 3x$

3. (4 points) Solve the initial value problem: $y'' - 5y' - 6y = 0; \quad y(0) = 1, y'(0) = -1$

Char eqn: $r^2 - 5r - 6 = 0$

$(r-6)(r+1) = 0$

$r = 6, \; r = -1$

$y_1 = e^{6x}, \; y_2 = e^{-x}$

$y(x) = c_1 e^{6x} + c_2 e^{-x}$

$y(0) = 1 \Rightarrow c_1 + c_2 = 1$

$y'(x) = 6c_1 e^{6x} - c_2 e^{-x}$

$y'(0) = -1 \Rightarrow 6c_1 - c_2 = -1$

$c_1 + c_2 = 1$

$6c_1 - c_2 = -1$

$7c_1 = 0$

$c_1 = 0$

$c_2 = 1$

$y(x) = e^{-x}$