Math 240 - Quiz 7

October 19, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 24.

1. (5 points) Find the general solution: $y'' - y = 8xe^x + 2e^x$

Homo. eqn: $y'' - y = 0$

$r^2 - 1 = 0$

$r = \pm 1$

$y_c(x) = c_1 e^x + c_2 e^{-x}$

Non Homo. eqn:

$g(x) = (8x+2)e^x$

$y_p(x) = x^s (Ax+B)e^x$

Must choose $s = 1$

so that

$y_p(x) = (Ax^2 + Bx)e^x$

$y_p'(x) = (2Ax+B)e^x + (Ax^2+Bx)e^x$

$y_p''(x) = 2Ae^x + (2Ax+B)e^x + (2Ax+B)e^x + (Ax^2+Bx)e^x$

$y_p''(x) - y_p(x) = 8xe^x + 2e^x$

$\Downarrow$

$2Ae^x + 2Axe^x + Be^x + 2Axe^x + Be^x + Ax^2e^x + Bxe^x - Ax^2e^x - Bxe^x$

$= 4Axe^x + (2A+2B)e^x$

$= 8xe^x + 2e^x$

$\Downarrow$

$4A = 8 \quad A = 2$

$2A + 2B = 2 \quad B = -1$

$y_p(x) = (2x^2 - x)e^x$

$y(x) = c_1 e^x + c_2 e^{-x} + (2x^2 - x)e^x$

Turn over.

2. (5 points) Find the general solution: $y'' + 3y' + 2y = x^2 + e^{-2x}$

Homo. eqn: $y'' + 3y' + 2y = 0$

$r^2 + 3r + 2 = 0$

$(r+2)(r+1) = 0$

$r = -2, \; r = -1$

$y_c(x) = c_1 e^{-2x} + c_2 e^{-x}$

Non Homo #1: $g(x) = x^2$

$y_{P_1}(x) = x^s (Ax^2 + Bx + C)$

$s = 0$

$y_{P_1}(x) = Ax^2 + Bx + C$

$y'_{P_1}(x) = 2Ax + B$

$y''_{P_1}(x) = 2A$

Substitute...

$2A + 3(2Ax + B) + 2(Ax^2 + Bx + C)$

$= 2Ax^2 + (6A + 2B)x + (2A + 3B + 2C)$

$= x^2 \Rightarrow$

$2A = 1$

$6A + 2B = 0$

$2A + 3B + 2C = 0$

$A = \frac{1}{2}, \; B = -\frac{3}{2},$

$C = \frac{1}{2}\left(-1 + \frac{9}{2}\right) = \frac{7}{4}$

$y_{P_1}(x) = \frac{1}{2}x^2 - \frac{3}{2}x + \frac{7}{4}$

Non Homo #2: $g(x) = e^{-2x}$

$y_{P_2}(x) = x^s A e^{-2x}$

$s = 1$

$y_{P_2}(x) = Axe^{-2x}$

$y'_{P_2}(x) = Ae^{-2x} - 2Axe^{-2x}$

$y''_{P_2}(x) = -2Ae^{-2x} - 2Ae^{-2x} + 4Axe^{-2x}$

Substitute...

$-4A + 4Ax + 3(A - 2Ax) + 2(Ax)$

$= -A = 1 \Rightarrow A = -1$

$y_{P_2}(x) = -xe^{-2x}$

$y(x) = c_1 e^{-2x} + c_2 e^{-x} - xe^{-2x} + \frac{1}{2}x^2 - \frac{3}{2}x + \frac{7}{4}$