

Math 240 - Quiz 8

October 26, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Find the general solution of $y'' - 2y' + y = \frac{e^t}{t}$.

Homo. eqn: $y'' - 2y' + y = 0$

$$r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0$$

$$r = 1 \text{ (multiplicity 2)}$$

$$y_c(t) = c_1 e^t + c_2 t e^t$$

NonHomo. eqn:

$$g(t) = \frac{e^t}{t}$$

USE VARIATION OF PARAMETERS

$$W = \begin{vmatrix} e^t & t e^t \\ e^t & e^t + t e^t \end{vmatrix}$$

$$= e^{2t}$$

$$v_1(t) = \int \frac{-\left(\frac{e^t}{t}\right) t e^t}{e^{2t}} dt$$

$$= \int -1 dt = -t$$

$$v_2(t) = \int \frac{\left(\frac{e^t}{t}\right) e^t}{e^{2t}} dt$$

$$= \int \frac{1}{t} dt = \ln|t|$$

$$y_p(t) = -t e^t + \ln|t| t e^t$$

$$y(t) = c_1 e^t + c_2 t e^t + t e^t \ln|t|$$