Math 240 - Quiz 9

November 16, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find the inverse Laplace transform of each function.

(a) $F(s) = \dfrac{3s+5}{s^2+9} = 3 \cdot \dfrac{s}{s^2+9} + \dfrac{5}{3} \cdot \dfrac{3}{s^2+9}$

$$f(t) = 3\cos 3t + \frac{5}{3}\sin 3t$$

(b) $F(s) = \dfrac{1}{s(s-1)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s-1} + \dfrac{C}{s+3}$

Cover-up $\Rightarrow$ $A = -\frac{1}{3}$, $B = \frac{1}{4}$, $C = \frac{1}{12}$

$$f(t) = -\frac{1}{3} + \frac{1}{4}e^{t} + \frac{1}{12}e^{-3t}$$

2. (5 points) Use Laplace transforms to solve the IVP.

$$x'' + 16x = \cos 4t, \quad x(0) = 0,\ x'(0) = 1$$

$$s^2X - s\cdot 0 - 1 + 16X = \frac{s}{s^2+16}$$

$$(s^2+16)X = \frac{s}{s^2+16} + 1$$

\#22 \#7

$$X = \frac{s}{(s^2+16)^2} + \frac{1}{s^2+16} = \frac{1}{8}\frac{8s}{(s^2+16)^2} + \frac{1}{4}\frac{4}{s^2+16}$$

$$x(t) = \frac{1}{8}t\sin 4t + \frac{1}{4}\sin 4t$$