Math 240 - Test 2

October 12, 2023

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand (showing work), unless otherwise specified.

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1. (10 points) Assume $x > 0$ and solve the following initial value problem.

$$y' + \frac{4}{x}y = x^3y^2, \quad y(1) = 1/2$$

2. (4 points) Explain why the functions $y_1(x) = x^2+1$, $y_2(x) = x^2+3x$, and $y_3(x) = 1-3x$ are linearly **dependent**.

3. (8 points) Find two linearly independent solutions of $y'' + 3y' - 18y = 0$. Use the Wronskian to show that your solutions are indeed independent.

4. (10 points) Solve the initial value problem.

$$y''' + 2y' = 0; \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 4$$

5. (12 points) Consider the equation $(x^2 - 1)y'' - 6xy' + 12y = 0$.

(a) The functions $y_1(x) = 1 + 6x^2 + x^4$ and $y_2(x) = x + x^3$ are solutions. Choose either one of them and verify that it is a solution.

(b) Find another solution (other than $y(x) \equiv 0$).

(c) Without using the Wronskian, argue that y_1 and y_2 are linearly independent.

(d) Should we expect a unique solution satisfying $y(1) = 2, y'(1) = 5$? Explain your reasoning.

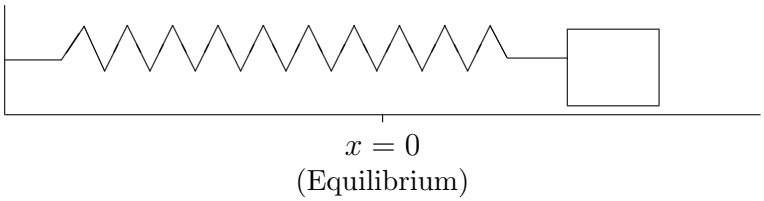
(e) Use what you've learned in parts (a) and (c) to find the solution of the IVP $(x^2 - 1)y'' - 6xy' + 12y = 0;\ y(0) = 2,\ y'(0) = 5$.

6. (10 points) Assuming $x > 0$, find the general solution of the Cauchy-Euler equation $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = 0$.

7. (6 points) The general solution of a homogeneous, constant-coefficient, linear differential equation is $y(x) = c_1e^{2x} + c_2xe^{2x} + c_3x^2e^{2x}$. Find such an equation.

8. (4 points) A mass-spring system is described by the equation $mx'' + bx' + kx = 0$. What must be true if the system is underdamped? Describe the motion of the mass in such a case.

9. (16 points) A 4-kg mass is attached to a spring with spring constant 17 N/m. The damping constant for the system is 4 N-sec/m. The mass is moved 3 m to the **right** of equilibrium (stretching the spring) and pushed to the **left** with a speed of 9.5 m/sec. Find the equation of motion. Write your solution in terms of a single sine or cosine with a phase shift.



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The following problems make up the take-home portion of the test. These problems are due October 17, 2023. You must work on your own.

10. (5 points) Solve: $x^2y' = xy + x^2e^{y/x}$

11. (7 points) Find the general solution. You may use a computer algebra system to factor the characteristic polynomial.

$$y^{(7)} - 3y^{(6)} + 2y^{(5)} - 6y^{(4)} + y''' - 3y'' = 0$$

12. (8 points) Solve: $y^3 y'' = 1$