Math 240 - Final Exam B

Name _____

December 14, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

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1. (4 points) Use any method to find the general solution of $y''' + 2y'' + 4y' = 0$.

2. (8 points) Use any method to find the general solution of $y'' - 2y' + y = 8e^t$.

3. (10 points) In this problem, you will find five (5) ordinary differential equations. Each equation has a specific name or can be described by a word, phrase, or short sentence. For each equation, write that name or description, and then write a sentence describing a solution method. Be brief, but specific, when describing your solution method.

(a) $\dfrac{1}{x}\dfrac{dy}{dx} - \dfrac{2y}{x^2} = x\cos x, \quad x > 0$

(b) $3xy^2\dfrac{dy}{dx} + y^3 = x^3$

(c) $y'' - 4y' + 4y = t^3e^{2t}; \quad y(0) = 0,\ y'(0) = 0$

(d) $(x + xy^2)\,dx + x^2y\,dy = 0$

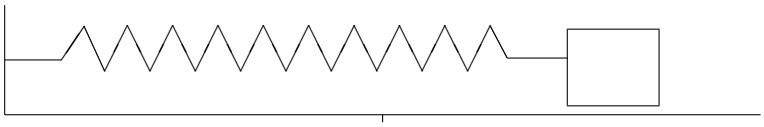
(e) $x'' - tx' + x = 0; \quad x(0) = 0,\ x'(0) = 0$

4. (16 points) Choose any two of the equations from problem 3 and solve each by using the solution method that you described above.

   (a) First problem:

(b) Second problem:

5. (12 points) A 1-kg mass is attached to a spring with spring constant $k = 25\,\text{N/m}$. The damping constant for the system is $b = 6\,\text{N-sec/m}$. The mass is moved 1 m to the right of equilibrium (stretching the spring) and released from rest. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



$x = 0$
(Equilibrium)

Follow-up: After the mass passes through equilibrium for the first time, it will very soon reach its farthest point to the left of equilibrium. When will it reach that point?