Math 240 - Assignment 1

August 21, 2025

Name ____________________

Score ______________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due August 28.

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1. Show (by substitution) that $y(x) = \dfrac{c \sin x}{x^2}$ is a solution of $x^2y'' + 4xy' + (x^2+2)y = 0$ for any constant c.

2. Assume that the unknown functions $y_1(x)$ and $y_2(x)$ both individually satisfy the equation $x^2y''+4xy'+(x^2+2)y = 0$. Show (by substitution) that $y(x) = ay_1(x)+by_2(x)$ is a solution for any constants a and b.

3. By guess and check, determine the solution of the equation $|y'|+|y| = 0$. Then explain how you can be certain that your solution is the only solution.

4. Suppose that you are doing research that involves the atmospheric pressure p at altitude a. Explain why the model
$$\frac{dp}{da} = kp$$
seems reasonable. (What sign does k have?) Then show (by substitution) that $p(a) = p_0e^{ka}$ is a solution, where $p_0 = p(0)$.

5. Classify the differential equation by saying whether it is ordinary or partial, linear or nonlinear. Also give its order and name the dependent and independent variables.
$$\frac{d^2u}{dw^2} + 5\left(\frac{du}{dw}\right)^3 - 4u = w$$

6. Is the following ordinary differential equation linear or nonlinear? Explain how you know. Then verify (by substitution) that $y = \ln(x + C)$ is a solution for any constant C. Finally, determine the constant C so that $y(0) = 0$.
$$e^y y' = 1$$

7. Solve the initial value problem: $\dfrac{dy}{dx} = x^2 \sin(\pi x), \quad y(1/2) = 2.$

8. Write a differential equation that models the problem situation:

   In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain disease is proportional to the product of the number who have the disease and the number who do not.

Turn over.

9. Read the problem situation below. Write a differential equation having $y = g(x)$ as one of its solutions.

The line normal to the graph of g at (x, y) passes through the point $(x/3, 1)$.

10. Solve the initial value problem: $\dfrac{dy}{dx} = \dfrac{x^3}{\sqrt{x^4 + 15}}, \quad y(1) = 5.$