Math 240 - Assignment 2

August 28, 2025

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 4.

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1. Suppose the population $P(t)$ (in thousands) of a certain species at time t satisfies the equation
$$\frac{dP}{dt} = P(P-1)(2-P).$$
Construct a slope field (use technology) to answer the following questions.
   (a) If the initial population is 4000, will the population increase or decrease? Quickly or slowly?
   (b) If the initial population is 4000, what is the limiting population?
   (c) If the initial population is 1500, what is the limiting population?
   (d) If the initial population is 2000, what will happen to the population?
   (e) What will happen to the population if the initial population is less than 1000?

2. Analyze the initial value problem to determine which one of these applies.
   (A) A solution exists, but it is not guaranteed to be unique.
   (B) There is a unique solution.
   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.
$$\frac{dy}{dx} = 3x - \sqrt[3]{y-1}, \quad y(2) = 1$$

3. Analyze the initial value problem to determine which one of these applies.
   (A) A solution exists, but it is not guaranteed to be unique.
   (B) There is a unique solution.
   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.
$$y' = \sqrt{xy}, \quad y(1) = 0$$

Turn over.

4. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.

   (B) There is a unique solution.

   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.

$$(y')^2 - xy' + y = 0, \quad y(2) = 1$$

5. Use Euler's method (by hand) with $h = 0.1$ to approximate $y(1.3)$.

$$\frac{dy}{dx} = x\sqrt{y}, \quad y(1) = 4.$$

   Follow-up: Use technology with $h = 0.01$ to approximate $y(1.3)$.

6. Solve the initial value problem: $\dfrac{dy}{dx} = y^2 - 4, \quad y(0) = -2$. (You'll need to use a partial fraction decomposition, and it will be possible to give an explicit solution.)

7. A cake at 300° F is removed from the oven and placed into a room where the ambient temperature is 70° F. After 3 minutes the cake has cooled to 200° F. Set up and solve the differential equation that gives the temperature of the cake at time t ($t \geq 0$). When will the cake be almost at room temperature? (Use Newton's law of cooling.)

8. Solve the initial value problem: $x^2y' = y - xy, \quad y(-1) = -1.$

9. Find the general solution: $\dfrac{dx}{dy} = \dfrac{1 + 2y^2}{y \sin x}$