

Math 240 - Assignment 3

September 4, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 11.

1. Solve the separable equation $\frac{dy}{dx} = 3y$ by treating it as a first-order linear equation.
2. Find the general solution: $x \frac{dy}{dx} - 4y = x^6 e^x$.
3. Solve the initial value problem: $y' + 2xy = x, \quad y(0) = -3$.
4. Solve the initial value problem by treating x as the dependent variable and y as the independent variable (so that the equation is linear in x).

$$\frac{dy}{dx} = \frac{1}{x + y^2}, \quad y = 0 \text{ when } x = -2$$

5. A large tank initially contains 300 gal of a salt solution in which 50 lb of salt are dissolved. A salt solution containing 2 lb of salt per gallon enters the tank at 3 gal/min and is uniformly mixed. The mixed solution leaves the tank at 2 gal/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up and solve the appropriate initial value problem to determine $A(t)$. How much salt is in the tank when the volume is 320 gal?
6. Suppose that a falling object is subject to two forces: the force of gravity and a force of air resistance proportional to the object's velocity. In such a case, Newton's 2nd law gives

$$m \frac{dv}{dt} = -mg - bv, \quad v(0) = v_0,$$

where m is the mass of the object, v is the velocity at time t , g is the acceleration due to the force of gravity, and b is the constant of proportionality associated with the force of air resistance. Solve the initial value problem to find $v(t)$. Then compute $\lim_{t \rightarrow \infty} v(t)$ and explain what the result means.

7. Solve: $(\cos x \sin x - xy^2) dx + y(1 - x^2) dy = 0$
8. Solve: $(x + \tan^{-1} y) dx + \frac{x + y}{1 + y^2} dy = 0$
9. Consider the equation $-y^2 dx + (x^2 + xy) dy = 0$. Show that the equation is NOT exact. Then multiply both sides of the equation by $(x^2 y)^{-1}$, and show that the new equation is exact. Solve the new equation.