Math 240 - Assignment 4

Name _____

September 18, 2025

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 23.

---

1. Find the general solution: $\dfrac{dy}{dx} = \dfrac{2y}{x} - x^2y^2$

2. Find the general solution: $(y^2 - xy)\,dx + x^2\,dy = 0$

3. Here is an example of a problem involving a *pursuit curve*: Criminals are in a boat at the point $(1, 0)$ when the police (at the origin) shine a spotlight on them. The criminals immediately evade the police by moving counter-clockwise at a $45°$ angle away from the light beam. Of course, the police will instantaneously readjust the light, and the criminals will continue to evade. The criminal's path will satisfy the initial value problem
$$\frac{dy}{dx} = \frac{y + x}{x - y}, \quad y(1) = 0.$$
Solve the initial value problem.

4. Use the substitution $u = x + y + 2$ to solve the initial value problem.
$$\frac{dy}{dx} = (x + y + 2)^2, \quad y(0) = -1$$

5. Find the general solution by using the techniques of section 1.6: $xy'' + y' = 4x$

6. Find the general solution by using the techniques of section 1.6: $y'' + y = 0$

7. Consider the equation $xy'' + 5y' = 0$ when $x > 0$.
   (a) Find the general solution by using the techniques of section 1.6.
   (b) Separately verify that both $y_1(x) \equiv 1$ and $y_2(x) = \dfrac{1}{x^4}$ are solutions.
   (c) Compute the Wronskian of y_1 and y_2 and conclude that these functions are linearly independent on $(0, \infty)$.
   (d) Find the particular solution that satisfies $y(1) = 2$ and $y'(1) = 4$.

8. Consider the following ODE: $yy'' + (y')^2 = 0$.
   (a) Verify that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions.
   (b) Show that $y_1(x) + y_2(x)$ is not a solution.
   (c) Why should you not expect the sum of solutions to be a solution?

9. Solve the initial value problem: $y'' - y' - 12y = 0, \quad y(0) = 2,\ y'(0) = -5$

10. Find the general solution: $y^{(4)} - y'' = 0$