

Math 240 - Test 1
September 11, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand (showing work), unless otherwise specified.

1. (4 points) Consider the differential equation $\frac{dy}{dx} = 2xy$.

(a) Steve “solved” the equation as follows:

$$dy = 2xy \, dx \implies \int dy = \int 2xy \, dx \implies y = x^2y + C,$$

and then he solved for y . Explain why Steve is all wrong.

(b) Without solving the equation, explain how Steve should have solved it.

2. (6 points) State whether each equation is ordinary or partial, linear or nonlinear, and give its order.

(a) $(1 - x)y'' - 4xy' + 5y = \cos x$

(b) $x \frac{d^3y}{dx^3} - 2 \left(\frac{dy}{dx} \right)^4 + y = 0$

(c) $10t \frac{\partial^2 u}{\partial x^2} = x \frac{\partial u}{\partial t}$

(d) $x^2 \frac{d^2x}{dt^2} + 5x \frac{dx}{dt} + 6t = 1$

3. (5 points) The equation $y'' - y = 0$ has the general solution $y(x) = c_1 e^x + c_2 e^{-x}$, where c_1 and c_2 are arbitrary constants. Find the particular solution that satisfies the initial conditions $y(0) = 3$ and $y'(0) = 8$.

4. (12 points) Analyze each initial value problem to determine which one of these applies.

Do not attempt to solve the equations.

(A) A solution exists, but it is not guaranteed to be unique.

(B) There is a unique solution.

(C) A solution is not guaranteed to exist.

Be sure to show work or explain.

(a) $\left(\frac{dy}{dx}\right)^2 + x^2 = y, \quad y(1) = 1$

(b) $(x - 5)\frac{dy}{dx} + (x^2 - 4)y = |x|, \quad y(0) = 5$

(c) $\frac{dy}{dx} = \frac{\sqrt[3]{x - y}}{x^2 + 1}, \quad y(4) = 4$

5. (9 points) Consider the equation $\frac{dy}{dx} = (xy - x)^2$.

(a) Find the general solution.

(b) Show that your general solution cannot satisfy $y(0) = 1$.

(c) Find a singular solution satisfying $y(0) = 1$.

6. (5 points) Recall that Newton's 2nd Law states that the acceleration (dv/dt) of an object is proportional to the sum of forces acting on the object. A falling object is acted on by a downward force equal to its weight, W , and an upward force (air resistance) that is proportional to its velocity. Write a differential equation for the object's velocity.

7. (8 points) Use Euler's method with $h = 0.1$ to estimate $y(0.4)$.

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 0$$

8. (12 points) Solve the initial value problem: $\frac{dN}{dt} + \frac{10N}{2t+3} = 8, \quad N(0) = 0 \quad (t \geq 0)$

9. (12 points) Find a function $N(x, y)$ so that the equation is exact. Then using that $N(x, y)$, find the general solution.

$$(x + y^2 \sin x + x^2 y - x e^{2y}) dx + N(x, y) dy = 0$$

10. (9 points) Show (or explain) that this equation is NOT separable, NOT linear, and NOT exact.

$$\frac{dy}{dx} + x^2 y - e^x y^3 = 0$$

Follow-up question: Does the equation fit into any type we have studied. If so, which?

Intentionally blank.

The following problems are take-home problems. They are due September 16. You must work on your own.

11. (3 points) A tank is initially filled with 1000 gal of a salt solution that has a salt concentration of 0.002 lb/gal. A salt solution containing 0.25 lb of salt per gallon enters the tank at 8 gal/min and is uniformly mixed. The mixed solution leaves the tank at 6 gal/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up the appropriate initial value problem to determine $A(t)$. DO NOT SOLVE.

12. (9 points) Solve the initial value problem: $x \frac{dy}{dx} - (1+x)y = xy^2, \quad y(1) = 1$

13. (6 points) Suppose that the amount of a substance decreases at a rate that is inversely proportional to the amount present. If 12 units of the substance are present initially and 8 units are present after 2 days, how long will it take the substance to disappear?