Math 240 - Test 2

October 9, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand, unless otherwise specified.

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1. (12 points) Solve: $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

2. (12 points) Solve: $\dfrac{dy}{dx} - y = e^x y^2$

3. (15 points) Consider the equation $(x-1)y'' - xy' + y = 0$.

(a) Verify that $y_1(x) = x$ and $y_2(x) = e^x$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on any interval not containing $x = 1$.

(c) Now consider the nonhomogeneous equation $(x-1)y'' - xy' + y = 2x - x^2$. Verify that $y_p(x) = 2 + x^2$ is a solution.

(d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $(x-1)y'' - xy' + y = 2x^2 - 4x;\ y(0) = 3,\ y'(0) = 9$.

(e) Is your solution in part (d) unique? Explain.

4. (8 points) Solve the following initial value problem.

$$y'' + y' - 12y = 0; \quad y(0) = 3,\ y'(0) = 23$$

5. (8 points) Find the general solution: $y^{(5)} + 7y^{(3)} = 0$

6. (8 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

   (a) $5x'' + 18x = 0$

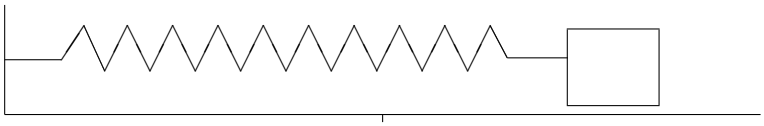
   (b) $x(t) = -3e^{-5t/7} - 13e^{-3t/7}$

   (c) $3x'' + 30x' + 75x = 0$

   (d) $x(t) = 15e^{-t/6}\cos(2t - \frac{\pi}{8})$

7. (4 points) Let $y_1(x) = 5x$, $y_2(x) = 2x^2 - 10x + 12$, and $y_3(x) = x^2 + 6$. Show that y_1, y_2, and y_3 are linearly dependent.

8. (15 points) A 1-kg mass is attached to a spring with spring constant 5 N/m. The damping constant for the system is 2 N-sec/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and pushed to the left at 1 m/sec. Find the equation of motion. Write your solution in terms of a single sine or cosine with a phase shift.



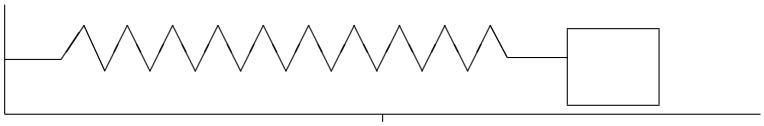
$x = 0$
(Equilibrium)

The following problems make up the take-home portion of the test. These problems are due October 14, 2025. You must work on your own.

9. (6 points) Find the general solution: $2x^2y'' - 10xy' + 68y = 0$

10. (6 points) Solve: $xy'' - 2y' = x^3 \sin 3x$

11. (6 points) A 8-kg mass is attached to a spring with spring constant 1 N/m. The damping constant for the system is 6 N-sec/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and pushed to the left at $\frac{1}{4}$ m/sec. Find the equation of motion. Then show that the mass never passes through equilibrium.



$x = 0$
(Equilibrium)