Math 240 - Final Exam A

Name _____

December 5, 2025

Score _____

Show all work to receive full credit. You must work individually. This test is due December 11. All integration must be done by hand (showing work).

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1. (10 points) A large tank initially contains 1000 L of a salt solution in which 0.5 kg of salt are dissolved. A salt solution containing 0.02 kg of salt per liter enters the tank at 4 L/min and is uniformly mixed. The mixed solution leaves the tank at 2 L/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up and solve the appropriate initial value problem to determine $A(t)$. When will the concentration of salt in the tank reach 0.01 kg/L?

2. (10 points) Use Laplace transform methods to solve the following equation.

$$ty'' + (2t - 3)y' + 2y = 0; \quad y(0) = 0, \quad y'(0) = 0$$

3. (10 points) Solve the following one-dimensional heat equation with Dirichlet boundary conditions. Do not derive the solution method–just use the result we derived in class. (See Theorem 1 on page 593.)

$$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 10, \quad t \geq 0,$$

$$u(0,t) = u(10,t) = 0,$$

$$u(x,0) = 100 - 10x, \quad 0 \leq x \leq 10$$