# Math 240 - Final Exam B       Name _____

December 11, 2025                                    Score _____

**Show all work to receive full credit**. Supply explanations where necessary.

___

1. (10 points) According to Newton's law of cooling, the temperature $T$ at time $t$ of an object cooling in a medium of constant temperature $M$ is described by the differential equation

$$\frac{dT}{dt} = k(M - T),$$

   where $k$ is some constant.

   (a) Solve the differential equation.

   (b) An object at 120°F is moved into a large room with an ambient temperature of 72°F. The object cools to 100°F in 6 min. Use your result from part (a) to find a formula for the temperature of the object at time $t$.

   (c) When will the object reach 76°F?

2. (10 points) The vector force $M(x,y)\,\hat{\imath}+N(x,y)\,\hat{\jmath}$ is a *conservative* force if the differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$ is an exact differential equation.

Consider the equation $(y\cos x + y^2)\,dx + (\sin x + 2xy - 2y)\,dy = 0$.

(a) Show that the equation is exact.

(b) When a force is conservative, we can find a *potential function* by solving the corresponding exact differential equation. Solve the exact equation above.

3. (10 points) Solve the initial value problem:  $y' + y \cot x = 2x \csc x, \quad y(\pi/2) = \pi^2$

4. (10 points) Find the general solution of each equation.

(a) $y^{(5)} - 8y^{(3)} + 16y' = 0$

(b) $x^2 y'' + 9xy' + 17y = 0, \quad x > 0$

5. (10 points) Use undetermined coefficients to solve.

$$y'' - 3y' + 2y = 14\sin 2x - 18\cos 2x; \qquad y(0) = 4, \quad y'(0) = -2$$

6. (10 points) The equation
$$(1 - x^2)y'' - xy' + 4y = 0$$
is an example of Chebyshev's equation of the first kind.

(a) Find the complete recurrence relation for a power series solution centered at $x = 0$.

(b) One of the two linearly independent solutions is a polynomial of degree 2. Find it.

(c) Find the first four nonzero terms of the second solution.

7. (10 points) Use Laplace transforms to solve the initial value problem.

$$y'' + 2y' - 15y = 6e^{-2t}; \qquad y(0) = 1, \quad y'(0) = 2$$