

Math 240 - Assignment 1

August 20, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due August 27.

1. According to Torricelli's law, the height, h , of water draining under the force of gravity from a cylindrical tank is described by the equation $\frac{dh}{dt} = -k\sqrt{h}$, where k is a constant that depends on the cross-sectional area of the tank. Say whether the equation is linear or nonlinear, state its order, and say which variables are independent and dependent.
2. The differential equation $x'' + \omega^2 x = 0$ arises in applications involving undamped oscillations.
 - (a) State the order of the equation and say which variable is independent and which is dependent.
 - (b) Say whether the equation is linear or nonlinear and explain how you know.
 - (c) Verify by substitution that $x(t) = A \cos \omega t + B \sin \omega t$ is a solution for any arbitrary constants A and B .
 - (d) Use the solution from part (c) to find the particular solution that satisfies the initial conditions $x(0) = 2$ and $x'(0) = -1$.
3. Consider the differential equation $2xy \frac{dy}{dx} = 4x^2 + 3y^2$.
 - (a) State the order of the equation and say which variable is independent and which is dependent.
 - (b) Say whether the equation is linear or nonlinear and explain how you know.
 - (c) Without attempting to solve the equation, verify that $y^2 + 4x^2 = kx^3$ is a solution for any arbitrary constant k .
 - (d) Is the solution given in part (c) an explicit solution or an implicit solution?
 - (e) Use the solution from part (c) to find the particular solution that satisfies $y(1) = 5$.
4. A plant sprouts at time $t = 0$ and grows in such a way that the instantaneous rate of change of its height, h , is proportional to the average rate of change of its height over its lifetime. Write a differential equation that models the height of the plant.

5. The equation below is the *Black-Scholes-Merton equation*, which describes the price evolution of financial derivatives. (Black and Scholes received the 1997 Nobel Prize in Economics.) Classify the differential equation by saying whether it is ordinary or partial, linear or nonlinear. Also give its order and name the dependent and independent variables.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV - rS \frac{\partial V}{\partial S}$$

6. Find the general solution: $x \frac{dy}{dx} = \frac{1}{x-1}$

7. For which values of r is $y(x) = e^{rx}$ a solution of $y'' - 2y' - 15y = 0$?

8. The graph of $y = f(x)$ has the following property:

The normal line through the point (x, y) passes through the point $(1, 2)$.

Write a differential equation that describes the family of such curves.

9. Solve the initial value problem: $\frac{dy}{dx} = 3x\sqrt{x^2 + 4}, \quad y(0) = 4.$

10. Solve the initial value problem: $\frac{1}{x \sin(2x)} \frac{dy}{dx} = 1, \quad y(0) = 4.$