

Math 240 - Assignment 2

September 27, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 1.

1. For $x \geq 0$, consider the initial value problem $dy/dx = y^{1/3}$, $y(0) = 0$.
 - (a) Use separation of variables to find the general solution of the equation. Then find the particular solution of the IVP that follows from your general solution.
 - (b) Find a singular solution.
2. Based on our existence/uniqueness theorem, explain why you should not have expected a unique solution for the IVP in problem 1.
3. Analyze the initial value problem to determine which one of these applies.
 - (A) A solution exists, but it is not guaranteed to be unique.
 - (B) There is a unique solution.
 - (C) A solution is not guaranteed to exist.

Be sure to show work or explain.

$$\frac{dy}{dx} = \ln(1 + y^2), \quad y(0) = 0$$

4. Analyze the initial value problem to determine which one of these applies.
 - (A) A solution exists, but it is not guaranteed to be unique.
 - (B) There is a unique solution.
 - (C) A solution is not guaranteed to exist.

Be sure to show work or explain.

$$(y')^2 - xy' + y = 0, \quad y(2) = 1$$

5. Analyze the initial value problem to determine which one of these applies.
 - (A) A solution exists, but it is not guaranteed to be unique.
 - (B) There is a unique solution.
 - (C) A solution is not guaranteed to exist.

Be sure to show work or explain.

$$\frac{dy}{dx} = (x^2 + 1)(y - 2)^{4/5}, \quad y(1) = -2$$

Turn over.

6. For $-5 \leq x, y \leq 5$, construct a slope field (use technology) for the differential equation $\frac{dy}{dx} = x^2 - y$. Then solve the following problems.

- (a) Determine the slope of the solution curve passing through $(1, 1)$.
- (b) Consider the solution that satisfies $y(-3) = 2$. Would you expect that solution to have relative maximum and minimum values? Explain.
- (c) Consider the solution that satisfies $y(0) = -4$. Would you expect that solution to have relative maximum and minimum values? Explain.
- (d) Based on the available information, what would you expect any solution to do as x approaches ∞ ? Explain.
- (e) Give two reasons to argue that the given slope field cannot be for the equation $dy/dx = -y/x$.

7. Use Euler's method (by hand) with $h = 0.1$ to approximate $y(0.3)$.

$$\frac{dy}{dx} = y - x - 1, \quad y(0) = 1.$$

Follow-up: Use technology with $h = 0.01$ to approximate $y(0.3)$.

8. Use Euler's method (by hand) with $h = 0.25$ to approximate $y(2)$.

$$\frac{dy}{dx} = x + \ln(y), \quad y(1) = 2.$$

9. Assuming k is some unspecified constant, find the general solution: $\frac{dv}{dt} = -kv^2$

10. Solve the initial value problem: $2y \frac{dy}{dx} = \frac{x}{\sqrt{x^2 - 16}}, \quad y(5) = 2.$

11. Solve the initial value problem: $\sqrt{y} dx + (1 + x) dy = 0, \quad y(0) = 1.$

12. A boat's motor shuts off when its velocity is 40 ft/s. After 10 seconds, the boat's velocity has slowed to 20 ft/s. Assume that the boat's acceleration is proportional to the square of the velocity (see problem 9). How far does the boat travel in the first 60 seconds that its motor is off?