Math 240 - Assignment 4

February 19, 2026

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 24.

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1. The following initial value problem was on Test 1:

$$x\frac{dy}{dx} - y = 2x^2y, \quad y(1) = 1.$$

   (a) Solve the problem by treating the equation as separable.

   (b) Solve the problem by treating the equation as linear.

   (c) Solve the problem by treating the equation as Bernoulli. (Hint: You'll need to use $u = \ln y$, assuming $y > 0$.)

2. Here is an example of a problem involving a *pursuit curve*: Criminals are in a boat at the point $(1, 0)$ when the police (at the origin) shine a spotlight on them. The criminals immediately evade the police by moving counter-clockwise at a 45° angle away from the light beam. Of course, the police will instantaneously readjust the light, and the criminals will continue to evade. The criminal's path will satisfy the initial value problem

$$\frac{dy}{dx} = \frac{y + x}{x - y}, \quad y(1) = 0.$$

   Solve the initial value problem.

3. The graph of $y = f(x)$ has the following property:

   The line tangent to the graph of f at (x, y) passes through the point $(x/2, 0)$.

   (a) Write a differential equation that describes the family of such curves.

   (b) Solve your equation by treating it as separable.

   (c) Solve your equation by treating it as linear.

   (d) Solve your equation by treating it as homogeneous.

4. Use the substitution $u = x + y$ to solve $(x + y)y' = 1$. (The equation will be separable after the substitution.)

5. Find the general solution: $xy^2y' = x^3 + y^3$.

6. Find the general solution: $x^2y' + 2xy = 5y^4$.

7. Solve the initial value problem: $y'' = y'e^y, \quad y(0) = 0, y'(0) = 1$.

8. Find the general solution: $xy'' + y' = 4x$.

Turn over.

9. It is easy to verify (don't bother) that $y_1(x) = x^2$ and $y_2(x) = x^3$ are two **different**, linearly independent solutions of the initial value problem

$$x^2y'' - 4xy' + 6y = 0; \quad y(0) = 0,\ y'(0) = 0.$$

   Explain why does this not contradict our existence/uniqueness theorem for linear equations?

10. Explain why the functions $y_1(x) = x^2 + 1$, $y_2(x) = x^2 + 3x$, and $y_3(x) = 1 - 3x$ are linearly **dependent**. Do not use the Wronskian.

11. Consider the following ODE: $yy'' + (y')^2 = 0$.

    (a) Verify that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions.

    (b) Show that $y_1(x) + y_2(x)$ is not a solution.

    (c) Why should you not expect the sum of solutions to be a solution?