

MTH 240 Assignment #5 key

①

① $y'' - 2y' = 0; y(0) = 3, y'(0) = -4$

$$r^2 - 2r = 0 \Rightarrow r(r-2) = 0 \Rightarrow r = 0, r = 2$$

$$\begin{aligned} y(x) &= c_1 + c_2 e^{2x} \\ y(0) &= 3 \Rightarrow c_1 + c_2 = 3 \\ y'(0) &= -4 \Rightarrow 2c_2 = -4 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} c_2 = -2 \\ c_1 = 5 \end{array}$$

$$y(x) = 5 - 2e^{2x}$$

② $6y'' - y' - 2y = 0$

$$6r^2 - r - 2 = 0 \Rightarrow (3r-2)(2r+1) = 0 \Rightarrow r = \frac{2}{3}, r = -\frac{1}{2}$$

$$y(x) = c_1 e^{\frac{2}{3}x} + c_2 e^{-\frac{1}{2}x}$$

③ $y''' - y'' - 9y' + 9y = 0$

$$r^3 - r^2 - 9r + 9 = 0 \Rightarrow r^2(r-1) - 9(r-1) = 0 \Rightarrow (r^2-9)(r-1) = 0$$
$$r = 3, r = -3, r = 1$$

$$y(x) = c_1 e^{3x} + c_2 e^{-3x} + c_3 e^x$$

(2)

$$(4) \quad 4y'' + 20y' + 25y = 0; \quad y(0) = -1, \quad y'(0) = -3$$

$$4r^2 + 20r + 25 = 0 \Rightarrow (2r + 5)^2 = 0 \Rightarrow r = -\frac{5}{2}, \quad r = -\frac{5}{2}$$

$$y(x) = c_1 e^{-\frac{5}{2}x} + c_2 x e^{-\frac{5}{2}x}$$

$$y(0) = -1 \Rightarrow c_1 = -1$$

$$y'(0) = -3 \Rightarrow -\frac{5}{2}c_1 + c_2 = -3 \Rightarrow \frac{5}{2} + c_2 = -3 \Rightarrow c_2 = -\frac{11}{2}$$

$$y(x) = -e^{-\frac{5}{2}x} - \frac{11}{2} x e^{-\frac{5}{2}x}$$

$$(5) \quad y''' + 2y'' + y' = 0; \quad y(0) = 2, \quad y'(0) = 0, \quad y''(0) = 1$$

$$r^3 + 2r^2 + r = 0 \Rightarrow r(r+1)^2 = 0 \Rightarrow r = 0, \quad r = -1, \quad r = -1$$

$$y(x) = c_1 + c_2 e^{-x} + c_3 x e^{-x}$$

$$y'(x) = -c_2 e^{-x} + c_3 e^{-x} - c_3 x e^{-x}$$

$$y''(x) = c_2 e^{-x} - c_3 e^{-x} - c_3 e^{-x} - c_3 x e^{-x}$$

$$y(0) = 2 \Rightarrow c_1 + c_2 = 2$$

$$y'(0) = 0 \Rightarrow -c_2 + c_3 = 0$$

$$y''(0) = 1 \Rightarrow c_2 - 2c_3 = 1$$

$$\left. \begin{array}{l} -c_2 + c_3 = 0 \\ c_2 - 2c_3 = 1 \end{array} \right\} \Rightarrow -c_3 = 1 \Rightarrow c_3 = -1 \Rightarrow c_2 = -1 \Rightarrow c_1 = 3$$

$$y(x) = 3 - e^{-x} - x e^{-x}$$

⑥ $y'' - 4y' + 13y = 0; \quad y(0) = -1, \quad y'(0) = 2$

$$r^2 - 4r + 13 = 0 \Rightarrow r^2 - 4r + 4 = -9 \Rightarrow (r-2)^2 = -9 \Rightarrow r-2 = \pm 3i$$

$$r = 2 \pm 3i$$

$$y(x) = c_1 e^{2x} \cos 3x + c_2 e^{2x} \sin 3x$$

$$y(0) = -1 \Rightarrow c_1 = -1$$

$$y'(0) = 2 \Rightarrow 2c_1 + 3c_2 = 2 \Rightarrow c_2 = \frac{4}{3}$$

$$y(x) = -e^{2x} \cos 3x + \frac{4}{3} e^{2x} \sin 3x$$

⑦ $y''' + 3y'' - 4y = 0$

$$r^3 + 3r^2 - 4 = 0$$

EASY TO SEE THAT $r=1$ IS A SOLUTION.

$$(r-1)(r^2 + 4r + 4) = 0$$

$$(r-1)(r+2)^2 = 0$$

$$r=1, \quad r=-2, \quad r=-2$$

$$\begin{array}{r} r^2 + 4r + 4 \\ r-1 \overline{) r^3 + 3r^2 + 0r - 4} \\ \underline{r^3 - r^2} \\ 4r^2 + 0r - 4 \\ \underline{4r^2 - 4r} \\ 4r - 4 \\ \underline{4r - 4} \\ 0 \end{array}$$

$$y(x) = c_1 e^x + c_2 e^{-2x} + c_3 x e^{-2x}$$

(4)

8) $x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 8y = 0$

Assume $x > 0$ AND LET $x = e^t$.

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 8y = 0$$

$$r^2 + 4r + 8 = 0 \Rightarrow (r+2)^2 = -4 \Rightarrow r = -2 \pm 2i$$

$$y(t) = c_1 e^{-2t} \cos 2t + c_2 e^{-2t} \sin 2t$$

Result... $y(x) = c_1 x^{-2} \cos(2 \ln x) + c_2 x^{-2} \sin(2 \ln x)$

9) $9x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 0$

Assume $x > 0$ AND LET $x = e^t$.

$$9 \frac{d^2 y}{dt^2} - 12 \frac{dy}{dt} + 4y = 0$$

$$9r^2 - 12r + 4 = 0 \Rightarrow (3r-2)^2 = 0 \Rightarrow r = \frac{2}{3}, r = \frac{2}{3}$$

$$y(t) = c_1 e^{\frac{2}{3}t} + c_2 t e^{\frac{2}{3}t}$$

Result... $y(x) = c_1 x^{2/3} + c_2 x^{2/3} \ln x$

10

$$r(r-5)^3 \underbrace{(r^2+6r+10)}_{(r+3)^2+1} = 0$$

$$r=0, r=5, r=5, r=5,$$

$$r = -3 \pm i$$

$$y(x) = c_1 + c_2 e^{5x} + c_3 x e^{5x} + c_4 x^2 e^{5x} \\ + c_5 e^{-3x} \cos x + c_6 e^{-3x} \sin x$$