

Math 240 - Assignment 9

October 29, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due November 5.

1. Consider the Cauchy-Euler equation $x^2y'' - xy' - 3y = 0$.

(a) Show that $x = 0$ is a regular singular point.

(b) Assume $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$ is a solution for some nonzero real number s . Differentiate, substitute, and equate coefficients as per our usual approach. (Because the series is not technically a power series, it is NOT appropriate to adjust the index when differentiating term by term.)

(c) Use a_0 as an arbitrary constant and assume $a_0 \neq 0$. What does your recurrence relation tell you when $n = 0$? (You should get what is called an *indicial equation*. That will give you two possible values for s . What are they?)

(d) One at a time, substitute your s -values into your recurrence relation from part (b) and solve for the coefficients a_n .

(e) Each s -value and the corresponding a_n 's gives you a solution for the original equation. What is the general solution?

2. Use the definition of the Laplace transform to find the transform of $f(t) = 2t$, $t \geq 0$.

3. Use the definition of the Laplace transform to find the transform of f .

$$f(t) = \begin{cases} 5, & 0 \leq t < 2 \\ 4, & t \geq 2 \end{cases}$$

4. Explain why the function $f(t) = 1$, $t \geq 0$ is of exponential order.

5. Explain why the function $f(t) = \cos t$, $t \geq 0$ is of exponential order.

6. Explain why the function $f(t) = e^{t^2}$, $t \geq 0$ is not of exponential order.

7. Find the inverse Laplace transform of each function. Show how you got your answer.

(a) $F(s) = \frac{3s+1}{s^2+4}$

(b) $F(s) = \frac{1}{s-5} - \frac{4}{s-3}$

(c) $F(s) = \frac{3}{s} - \frac{2}{s^4}$

Turn over.

8. Let $F(s) = \frac{4}{s(s+1)}$. Find the inverse Laplace transform by first computing the partial fraction decomposition (by hand).
9. Find the inverse Laplace transform of $F(s)$ by first computing the partial fraction decomposition. You may use technology to compute the partial fraction decomposition.

$$F(s) = \frac{s+1}{s^2(s+2)^3}$$