

Math 240 - Test 1
September 10, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand (showing work), unless otherwise specified.

1. (8 points) Consider the differential equation $\frac{dy}{dx} = y^2$.

(a) State the order of the equation and say which variable is independent and which is dependent.

(b) Say whether the equation is linear or nonlinear and explain how you know.

(c) Verify by substitution that $y(x) = \frac{1}{C - x}$ is a solution for any arbitrary constant C .

(d) Use the solution from part (c) to find the particular solution that satisfies $y(2) = 7$.

(e) Find (by observation) a singular solution of the differential equation.

2. (2 points) In modelling a problem situation, a researcher proposed the following differential equation:

$$\frac{d^2u}{dx^2} = 3x \frac{d^2u}{dy^2}.$$

Explain why this differential equation is a partial differential equation despite the fact that the researcher wrote the equation using the notation of ordinary derivatives.

3. (3 points) The graph of $y = f(x)$ has the following property:

The tangent line at the point (x, y) passes through the point $(-y, x)$.

Write a differential equation that describes the family of such curves.

4. (12 points) Using our existence/uniqueness theorems, analyze each initial value problem to determine which one of these applies. **Do not attempt to solve the equations.**

(A) A solution exists, but it is not guaranteed to be unique.

(B) There is a unique solution.

(C) A solution is not guaranteed to exist.

Be sure to show work or explain.

(a) $(x^2 + 1) \frac{dy}{dx} + 3xy = 6x, \quad y(1) = 7$

(b) $\frac{dy}{dx} = (x - 3)(y + 1)^{2/3}, \quad y(0) = -1$

(c) $\frac{dy}{dx} = \sqrt{x - y}, \quad y(2) = 2$

5. (6 points) An object gradually speeds up from rest in such a way that its acceleration over the first 10 seconds is given by

$$\frac{d^2x}{dt^2} = 0.12t^2 + 0.6t; \quad x(0) = 0, \quad x'(0) = 0,$$

where x is measured in feet. Find $x(5)$ and $x'(5)$.

6. (12 points) Solve the initial value problem: $x \frac{dy}{dx} - y = 2x^2y, \quad y(1) = 1.$

7. (12 points) Solve the initial value problem: $\frac{dy}{dx} - y = e^{-2x}, \quad y(0) = 1.$

8. (8 points) Use Euler's method with $h = 0.1$ to estimate $y(0.3)$.

$$\frac{dy}{dx} - y = e^{-2x}, \quad y(0) = 1$$

9. (12 points) The acceleration (dv/dt) of a certain sports car is proportional to the difference between its velocity and 250 km/h. The car can accelerate from rest to 100 km/h in 10 seconds ($10/3600$ hours). How long will it take for the car to go from rest to 200 km/h?

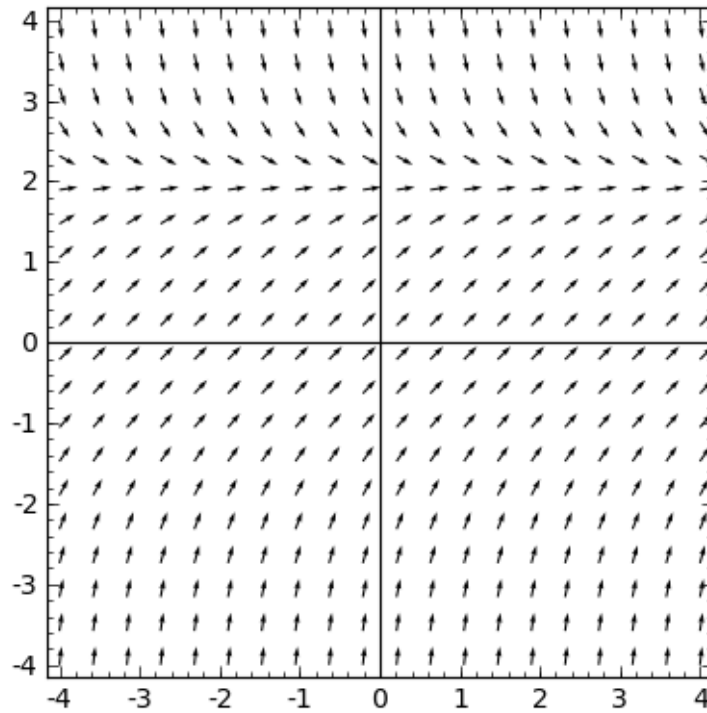
10. (11 points) Show that the equation IS NOT exact. Then determine the type of equation and find the general solution.

$$(y - 12x) dx + 3x dy = 0$$

11. (10 points) Show that the equation is exact. Then find the general solution.

$$\left(\frac{1}{x} + 2y^2x\right) dx + (2yx^2 - \cos y) dy = 0$$

12. (4 points) A direction field for $\frac{dy}{dx} = 1 - \frac{y^3}{8}$ is shown below. Use the direction field to solve the following problems.



- (a) What is the unique solution passing through $(-2, 2)$?
- (b) Suppose you have found a solution, $y(x)$, passing through a given point. What can you say about $\lim_{x \rightarrow \infty} y(x)$?