

Math 240 - Test 2

October 8, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand (showing work), unless otherwise specified.

1. (8 points) Find the general solution: $xy' = y + 2\sqrt{xy}; \quad x, y > 0$

2. (8 points) Solve the initial value problem: $3y^2y' + y^3 = e^{-x}, \quad y(0) = 2$

3. (4 points) Use a substitution to reduce the equation to first order. Stop once the equation is reduced. DO NOT SOLVE THE REDUCED EQUATION.

$$y'' = 2y(y')^3$$

4. (8 points) Solve the initial value problem: $y'' - 2y' - 15y = 0$; $y(0) = 2$, $y'(0) = 3$

5. (8 points) A homogeneous, constant-coefficient, linear differential equation has the following characteristic equation:

$$r^3(r + 2)(r - 3)(r^2 + 2r + 10) = 0.$$

Find the general solution of the original differential equation.

6. (4 points) Would you expect the initial value problem

$$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2; \quad y(1) = 2, y'(1) = 8$$

to have a unique solution? Explain your reasoning.

7. (9 points) Solve the homogeneous Cauchy-Euler equation assuming $x > 0$.

$$4x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} + y = 0$$

8. (15 points) Consider the equation $xy'' + 5y' = 0$, $x > 0$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = \frac{1}{x^4}$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Now consider the nonhomogeneous equation $xy'' + 5y' = 12x$, $x > 0$. Verify that $y_p(x) = x^2$ is a solution.

(d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $xy'' + 5y' = 12x$; $y(1) = 2$, $y'(1) = 10$.

(e) Is your solution in part (d) unique? Explain.

9. (12 points) Find the general solution: $y'' + 6y' + 10y = 5x + 8$.

10. (8 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

(a) $x(t) = \sqrt{6} \sin(4t + \pi)$

(b) $2x'' + 5x' + 3x = 0$

(c) $x(t) = 2e^{-2t} + 5te^{-2t}$

(d) $x'' + 8x' + 17x = 0$

11. (16 points) A 5.0-kg mass is attached to a spring with spring constant 11.3 N/m. The damping constant for the system is 1.0 N-sec/m. The mass is moved 2 m to the right of equilibrium (stretching the spring) and pushed to the right at 1 m/sec. (See the figure below.) Find the displacement at time t . Write your solution in terms of a single sine or cosine with a phase shift. When does the mass pass through equilibrium for the second time?

