Math 240 - Quiz 10

April 21, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 26.

1. (4 points) Use Laplace transform techniques to solve the initial value problem. You may use technology to compute any required partial fraction decompositions.

$$x'' + 4x' + 13x = te^{-t}; \quad x(0) = 0,\ x'(0) = 2$$

$$s^2 X(s) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{2} + 4\left(s X(s) - \cancel{x(0)}^{0}\right) + 13 X(s) = \frac{1}{(s+1)^2}$$

$$(s^2 + 4s + 13)\, X(s) - 2 = \frac{1}{(s+1)^2}$$

$$X(s) = \frac{\frac{1}{(s+1)^2} + 2}{\underbrace{s^2+4s+13}_{(s+2)^2+9}} = \frac{1}{50}\frac{s+98}{(s+2)^2+3^2} - \frac{1}{50}\frac{1}{s+1} + \frac{1}{10}\frac{1}{(s+1)^2}$$

$$= \frac{1}{50}\frac{s+2}{(s+2)^2+3^2} + \frac{1}{50}\frac{96}{(s+2)^2+3^2} - \frac{1}{50}\frac{1}{s+1} + \frac{1}{10}\frac{1}{(s+1)^2}$$

$$x(t) = \frac{1}{50}e^{-2t}\cos 3t + \frac{32}{50}e^{-2t}\sin 3t - \frac{1}{50}e^{-t} + \frac{1}{10}te^{-t}$$

Turn over.

2. (4 points) Use Laplace transform techniques to solve the system of initial value problems. You may use technology to compute any required partial fraction decompositions.

$$x' = x + 2y, \quad y' = x + e^{-t}; \quad x(0) = 0, \ y(0) = 0$$

$$X = \frac{2}{s-1} Y$$

$$sX(s) - x(0) = X(s) + 2Y(s) \qquad (s-1)X - 2Y = 0$$

$$sY(s) - y(0) = X(s) + \frac{1}{s+1} \qquad \left(-X + sY = \frac{1}{s+1}\right)(s-1)$$

$$\underbrace{\left[s(s-1) - 2\right]}_{(s+1)(s-2)} Y = \frac{s-1}{s+1} \Rightarrow Y = \frac{s-1}{(s-2)(s+1)^2}$$

$$X = \frac{2}{(s-2)(s+1)^2} = -\frac{2}{9}\frac{1}{s+1} + \frac{2}{9}\frac{1}{s-2} - \frac{2}{3}\frac{1}{(s+1)^2} \qquad X = \frac{2}{\cancel{s-1}} \cdot \frac{\cancel{s-1}}{(s-2)(s+1)^2}$$

$$Y = \frac{s-1}{(s-2)(s+1)^2} = -\frac{1}{9}\frac{1}{s+1} + \frac{1}{9}\frac{1}{s-2} + \frac{2}{3}\frac{1}{(s+1)^2}$$

$$x(t) = -\frac{2}{9}e^{-t} + \frac{2}{9}e^{2t} - \frac{2}{3}te^{-t}$$

$$y(t) = -\frac{1}{9}e^{-t} + \frac{1}{9}e^{2t} + \frac{2}{3}te^{-t}$$

3. (2 points) Find the inverse Laplace transform of $F(s) = \dfrac{1}{s(s^2-9)}$. You may use technology to compute any required partial fraction decompositions.

$$F(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-3} = \frac{-1/9}{s} + \frac{1/18}{s+3} + \frac{1/18}{s-3}$$

$$A = -\frac{1}{9}, \ B = \frac{1}{18}, \ C = \frac{1}{18}$$

$$f(t) = -\frac{1}{9} + \frac{1}{18}e^{-3t} + \frac{1}{18}e^{3t}$$