

Math 240 - Quiz 2

January 27, 2022

Name key

Score _____

This quiz is available in Canvas. It is due February 1.

1. (2 points) The quantity P is growing in such a way that $\frac{dP}{dt} = kP$. Find $P(25)$ if $P(0) = 10$ and $P(10) = 180$.

(a) 13746

(b) 18080

(c) 4500

(d) 29160

$$P(t) = P_0 e^{kt} = 10e^{kt}$$

$$P(10) = 180 = 10e^{10k} \Rightarrow k = \frac{\ln 18}{10}$$

$$P(25) = 10e^{25k} \approx \boxed{13746.15}$$

2. (1 point) Determine whether we should expect a unique solution, no solution, or more than one solution.

$$x \frac{dy}{dx} = (2y - 4)^{1/3}, \quad y(4) = 2$$

$$f(x,y) = \frac{(2y-4)^{1/3}}{x}$$

CONTINUOUS FOR $x \neq 0$

(a) We expect a unique solution.

(b) We expect a solution, but not necessarily a single solution.

(c) We expect no solution.

$$f_y(x,y) = \frac{2}{x(2y-4)^{2/3}}$$

CONT. FOR $x \neq 0$ AND $y \neq 2$

3. (1 point) Determine whether we should expect a unique solution, no solution, or more than one solution.

$$\frac{dy}{dx} + xe^x y = x^3 \cos x, \quad y(-1) = \sqrt{2}$$

(a) We expect a unique solution.

(b) We expect a solution, but not necessarily a single solution.

(c) We expect no solution.

$$f(x,y) = x^3 \cos x - xe^x y$$

$$f_y(x,y) = -xe^x$$

BOTH CONT. EVERYWHERE IN \mathbb{R}^2

4. (2 points) Solve. Then determine $y(2)$.

$$(\tan x) \frac{dy}{dx} = y, \quad y(\pi/2) = \pi/2$$

(a) 0.05482

(b) 1.4283

(c) 0.6060

(d) 1.4801

$$\frac{1}{y} dy = \cot x dx$$

$$y(\frac{\pi}{2}) = \frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}$$

$$\ln |y| = \ln |\sin x| + C$$

$$|y| = |\sin x| e^C \Rightarrow y = C \sin x$$

$$\frac{\pi}{2} \sin(2) \approx 1.428321$$

5. (2 points) Use Newton's law of cooling to solve: A cake is removed from an oven at 350°F and left to cool in a room at 70°F. After 30 minutes, the temperature of the cake is 150°F. When will the temperature be 110°F?

- (a) About 39 minutes
- (b) About 42 minutes
- (c) About 52 minutes
- (d) About 47 minutes

$$\frac{dT}{dt} = k(T - m) \Rightarrow$$

$$T(t) = M + Ce^{kt}$$

$$T(t) = 70 + Ce^{kt}$$

$$T(0) = 350 \Rightarrow C = 280$$

$$T(30) = 150 = 70 + 280e^{30k}$$

$$k = \frac{\ln(80/280)}{30}$$

$$110 = 70 + 280e^{kt}$$

$$t = \frac{\ln(40/280)}{k} \approx \boxed{46.60}$$

6. (2 points) Solve. Then determine $y(3)$.

$$x \frac{dy}{dx} = 2y + x^3 \cos x, \quad y(\pi) = 1$$

- (a) 2.2701
- (b) 3.4761
- (c) 2.1820
- (d) 1.2701

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \cos x$$

$$\mu(x) = e^{\int -\frac{2}{x} dx} = e^{-2 \ln|x|} = \frac{1}{x^2}$$

$$\frac{1}{x^2} y(x) = \int \cos x dx$$

$$\frac{1}{x^2} y(x) = \sin x + C$$

$$y(x) = x^2 \sin x + Cx^2$$

$$y(\pi) = 1 \Rightarrow 1 = C\pi^2 \Rightarrow C = \frac{1}{\pi^2}$$

$$y(x) = x^2 \sin x + \frac{1}{\pi^2} x^2$$

$$y(3) = 9 \sin 3 + \frac{9}{\pi^2}$$

$$\approx 2.18197$$