Math 240 - Quiz 3

February 3, 2022

Name key

Score _____

This quiz is available in Canvas. It is due February 8.

1. (2 points) Solve the initial value problem. Then find $y(1)$.

$$\frac{dy}{dx} + 2xy = x, \quad y(0) = -3$$

(a) 1.7876
(b) −9.0140
(c) −0.7876
(d) 1.2876

Linear...

$\mu(x) = e^{\int 2x\,dx} = e^{x^2}$

$e^{x^2} y(x) = \int x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$

$u = x^2$
$du = 2x\,dx$

$y(x) = \frac{1}{2} + Ce^{-x^2}$ $\quad y(0) = -3 \Rightarrow -3 = \frac{1}{2} + C$

$C = -\frac{7}{2}$

$y(x) = \frac{1}{2} - \frac{7}{2} e^{-x^2}$

$y(1) = \frac{1}{2} - \frac{7}{2} e^{-1} \approx -0.7876$

2. (2 points) Solve the initial value problem. Then find $y(2)$.

$$\frac{dy}{dx} = y + x^2, \quad y(0) = -2$$

(a) −9.1666
(b) −8.5625
(c) −10.0000
(d) −2.0000

$\frac{dy}{dx} - y = x^2$ Linear...

$\mu(x) = e^{\int -dx} = e^{-x}$

+	x^2	e^{-x}
−	$2x$	$-e^{-x}$
+	2	e^{-x}
−	0	$-e^{-x}$

$e^{-x} y(x) = \int x^2 e^{-x} dx$

$= e^{-x}(-x^2 - 2x - 2) + C$

$y(x) = Ce^x - x^2 - 2x - 2$

$y(0) = -2 \Rightarrow C = 0$

$y(x) = -x^2 - 2x - 2$

$y(2) = -4 - 4 - 2 = -10$

$A(t)$ = AMOUNT OF SALT AT TIME t

$V(t)$ = VOLUME AT TIME t = $100 + 2t$

3. (2 points) A large tank is partially filled with 100 gallons of fluid in which 10 lb of salt is dissolved. Brine containing 0.5 lb of salt per gallon is pumped into the tank at a rate of 6 gallons per minute. The well-mixed solution is then pumped out at a slower rate of 4 gallons per minute. Find the number of pounds of salt in the tank after 40 minutes.

(a) 87.63 lb
(b) 77.65 lb
(c) 64.38 lb
(d) 24.40 lb

$$\frac{dA}{dt} = 3 - \frac{4A}{100+2t}, \quad A(0) = 10$$

$$\frac{dA}{dt} + \frac{4}{100+2t} A = 3, \quad A(0) = 10$$

SOLUTION IS $A(t) = \dfrac{t^3 + 150t^2 + 7500t + 25000}{(t+50)^2}$

$$A(40) \approx 77.65$$

4. (2 points) Solve the initial value problem. Then assume $y \geq 0$ and find $y(0.25)$.

$$\underbrace{(\cos x \sin x - xy^2)}_{M}\,dx + \underbrace{y(1-x^2)}_{N}\,dy = 0, \qquad y(0) = 2$$

(a) 2.2952
(b) 1.5966
(c) 2.0497
(d) 4.4214

$\dfrac{\partial M}{\partial y} = -2xy = \dfrac{\partial N}{\partial x} \Rightarrow$ EQUATION IS EXACT.

$f_x = \cos x \sin x - xy^2 \Rightarrow f = \frac{1}{2}\sin^2 x - \frac{1}{2}x^2y^2 + g(y)$

$f_y = y(1-x^2) \Rightarrow f = \frac{1}{2}y^2 - \frac{1}{2}x^2y^2 + h(x)$

SOLUTION IS $f(x,y) = \frac{1}{2}\sin^2 x - \frac{1}{2}x^2y^2 + \frac{1}{2}y^2 = C$

$y(0) = 2 \Rightarrow C = 2$

$\sin^2 x - x^2y^2 + y^2 = 4$

$x = 0.25 \Rightarrow y = 4.2013777$

5. (2 points) Solve the initial value problem. Then find $y(2)$.

$$\frac{dy}{dx} + \frac{1}{x}y = xy^2, \quad y(1) = 2$$

(a) 2.0000
(b) 1.5000
(c) −1.0000
(d) −1.5000

BERNOULLI ...

$y^{-2}\dfrac{dy}{dx} + \dfrac{1}{x}y^{-1} = x$

$u = y^{-1}, \quad \dfrac{du}{dx} = -y^{-2}\dfrac{dy}{dx}$

$-\dfrac{du}{dx} + \dfrac{1}{x}u = x$

$\dfrac{du}{dx} - \dfrac{1}{x}u = -x$

LINEAR ...

$\mu(x) = e^{\int -\frac{1}{x}dx} = \dfrac{1}{|x|}$

$\dfrac{1}{|x|}u(x) = \int -\dfrac{x}{|x|}\,dx$

$\dfrac{1}{x}u = \int -1\,dx$

$= -x + C$

$u(x) = Cx - x^2$

$y(x) = \dfrac{1}{Cx - x^2}$

$y(1) = 2 \Rightarrow C = \frac{3}{2}$

$y(x) = \dfrac{1}{\frac{3}{2}x - x^2}$

$y(2) = \dfrac{1}{3-4} = -1$