Math 240 - Quiz 4

February 17, 2022

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due February 22.

1. (3 points) Consider the initial value problem $x^2y''-5xy'+5y=0$; $y(1)=4$, $y'(1)=2$.

   (a) Show that $y_1(x)=x^5$ and $y_2(x)=x$ solve the differential equation.

$y_1 = x^5$

$y_1' = 5x^4$

$y_1'' = 20x^3$

$x^2(20x^3) - 5x(5x^4) + 5x^5$

$= 20x^5 - 25x^5 + 5x^5$

$= 0$ ✓

$y_2 = x$

$y_2' = 1$

$y_2'' = 0$

$x^2(0) - 5x(1) + 5x$

$= -5x + 5x = 0$ ✓

   (b) Use the Wronskian to show that y_1 and y_2 are linearly independent for $x>0$.

$$W = \begin{vmatrix} x^5 & x \\ 5x^4 & 1 \end{vmatrix} = x^5 - 5x^5 = -4x^5 \neq 0 \text{ FOR } x>0$$

$W \neq 0$ FOR $x>0 \Rightarrow y_1$ & y_2 ARE LINEARLY INDEP FOR $x>0$

   (c) Find the solution of the initial value problem.

FROM (a) AND (b),

$y(x) = c_1x^5 + c_2x$.

$y'(x) = 5c_1x^4 + c_2$

$y(1)=4 \Rightarrow c_1+c_2=4$

$y'(1)=2 \Rightarrow 5c_1+c_2=2$

$4c_1 = -2$

$c_1 = -\frac{1}{2}$

$c_2 = \frac{9}{2}$

$\boxed{y(x) = \frac{9}{2}x - \frac{1}{2}x^5}$

2. (2 points) Solve: $y''+12y'+36y=0$; $y(0)=2$, $y'(0)=-3$

CHAR. EQN: $r^2+12r+36=0$

$(r+6)^2=0$

$r=-6,\ r=-6$

$y(x) = c_1e^{-6x} + c_2xe^{-6x}$

$y'(x) = -6c_1e^{-6x} + c_2e^{-6x} - 6c_2xe^{-6x}$

$y(0)=2 \Rightarrow c_1=2$

$y'(0)=-3 \Rightarrow -6c_1+c_2=-3$

$-12+c_2=-3$

$c_2=9$

Turn over.

$\boxed{y(x) = 2e^{-6x} + 9xe^{-6x}}$

3. (2 points) Find the general solution: $y^{(4)} + 3y^{(3)} + 2y'' = 0$

CHAR Eqn: $r^4 + 3r^3 + 2r^2 = 0$

$r^2(r+1)(r+2) = 0$

$r=0,\ r=0,\ r=-1,$
$r=-2$

$\{e^{0x}, xe^{0x}, e^{-1x}, e^{-2x}\}$

$y(x) = C_1 + C_2 x + C_3 e^{-x} + C_4 e^{-2x}$

4. (3 points) Consider the equation $y'' + 2y' - 15y = -32e^{-x}$.

(a) Show that $y_p(x) = 2e^{-x}$ is a particular solution.

$y_p(x) = 2e^{-x}$

$y_p'(x) = -2e^{-x}$

$y_p''(x) = 2e^{-x}$

$y_p''(x) + 2y_p'(x) - 15y_p(x)$

$= 2e^{-x} - 4e^{-x} - 30e^{-x} = -32e^{-x}$ ✓

(b) Find the general solution.

Homo eqn: $y'' + 2y' - 15y = 0$

CHAR Eqn: $r^2 + 2r - 15 = 0$

$(r+5)(r-3) = 0$

$r = -5,\ r = 3$

$y_h(x) = C_1 e^{-5x} + C_2 e^{3x}$

GENERAL SOLUTION IS

$y(x) = y_p(x) + y_h(x)$

OR

$y(x) = 2e^{-x} + C_1 e^{-5x} + C_2 e^{3x}$