Math 240 - Quiz 5

February 24, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 1.

1. (2.5 points) Find the general solution: $y^{(4)} + 8y'' + 16y = 0$

CHAR. EQN: $r^4 + 8r^2 + 16 = 0$

$(r^2+4)^2 = 0$

$(r^2+4)(r^2+4) = 0$

$r = \pm 2i \quad r = \pm 2i$

REPEATED COMPLEX CONJUGATE SOL'NS.

$\{e^{0x}\cos 2x,\ e^{0x}\sin 2x,$

$x e^{0x}\cos 2x,\ x e^{0x}\sin 2x\}$

$y(x) = c_1\cos 2x + c_2 \sin 2x + c_3 x\cos 2x + c_4 x \sin 2x$

2. (2.5 points) Solve the Cauchy-Euler equation: $x^2y'' - 3xy' + 4y = 0$

THE SUBSTITUTION $x = e^t$, $x > 0$,

gives

$y'' - 4y' + 4y = 0$

CHAR. EQN: $r^2 - 4r + 4 = 0$

$(r-2)(r-2) = 0$

$r = 2,\ r = 2$

$y(t) = c_1 e^{2t} + c_2 t e^{2t}$

RESUB $x = e^t$, $t = \ln x$

$y(x) = c_1 x^2 + c_2 x^2 \ln x$

Turn over.

3. (3 points) Solve: $y'' - 4y' + 7y = 0; \quad y(0) = 1, y'(0) = -1$

CHAR. EQN: $r^2 - 4r + 7 = 0$

$r^2 - 4r + 4 = -3$

$(r-2)^2 = -3$

$r = 2 \pm \sqrt{3}\,i$

$\{ e^{2x}\cos\sqrt{3}x,\ e^{2x}\sin\sqrt{3}x \}$

$y(x) = c_1 e^{2x}\cos\sqrt{3}x + c_2 e^{2x}\sin\sqrt{3}x$

$y(0) = 1 \Rightarrow c_1 = 1$

$y(x) = e^{2x}\cos\sqrt{3}x + c_2 e^{2x}\sin\sqrt{3}x$

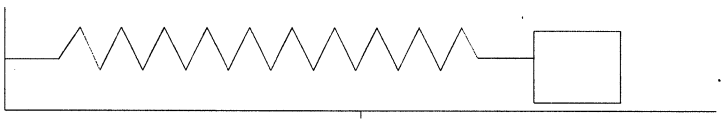
$y'(x) = 2e^{2x}\cos\sqrt{3}x - \sqrt{3}e^{2x}\sin\sqrt{3}x + 2c_2 e^{2x}\sin\sqrt{3}x + \sqrt{3}c_2 e^{2x}\cos\sqrt{3}x$

$y'(0) = -1 \Rightarrow 2 + \sqrt{3}c_2 = -1$

$c_2 = -\frac{3}{\sqrt{3}} = -\sqrt{3}$

$y(x) = e^{2x}\cos\sqrt{3}x - \sqrt{3}e^{2x}\sin\sqrt{3}x$

4. (2 points) A 1-kg mass is attached to a spring with spring constant 5 N/m. The damping constant for the system is 2 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and released from rest. Set up (but do not solve) the initial value problem that describes the motion. Say whether the motion is underdamped, overdamped, or critically damped.



$x = 0$
(Equilibrium)

$m = 1$
$k = 5$
$b = 2$

$x(0) = 1$
$x'(0) = 0$

$x'' + 2x' + 5x = 0;\ x(0) = 1,\ x'(0) = 0$

$2^2 - 4(1)(5) < 0 \Rightarrow$ UNDERDAMPED